

Single electron tunneling at large conductance: The semiclassical approach

G. Göppert^a and H. Grabert

Fakultät für Physik, Albert–Ludwigs–Universität, Hermann–Herder–Straße 3, 79104 Freiburg, Germany

Received 2 February 2000

Abstract. We study the linear conductance of single electron devices showing Coulomb blockade phenomena. Our approach is based on a formally exact path integral representation describing electron tunneling nonperturbatively. The electromagnetic environment of the device is treated in terms of the Caldeira-Leggett model. We obtain the linear conductance from the Kubo formula leading to a formally exact expression which is evaluated in the semiclassical limit. Specifically we consider three models. First, the influence of an electromagnetic environment of arbitrary impedance on a single tunnel junction is studied focusing on the limits of large tunneling conductance and high to moderately low temperatures. The predictions are compared with recent experimental data. Second, the conductance of an array of N tunnel junctions is determined in dependence on the length N of the array and the environmental impedance. Finally, we consider a single electron transistor and compare our results for large tunneling conductance with experimental findings.

PACS. 73.23.Hk Coulomb blockade; single-electron tunneling – 73.40.Gk Tunneling – 73.40.Rw Metal-insulator-metal structures

1 Introduction

Tunneling of electrons in nanostructures is strongly affected by Coulomb repulsion. In systems containing metallic tunnel junctions the interaction can be described by the charging energy [1] $E_C = e^2/2C$ expressed in terms of a geometrical capacitance C . For weak tunneling and temperatures well below E_C/k_B , tunneling is suppressed by the Coulomb blockade effect. This regime is well explored experimentally [2–5], and the phenomena observed can be explained theoretically [6–10] by means of perturbation theory in the tunneling strength which is characterized by the classical high temperature tunneling conductance G_T . This approach breaks down for conductances comparable to or even larger than the conductance quantum $G_K = e^2/h$. When using these devices, *e.g.* as highly sensitive electrometers [11], in detectors [12], or for thermometry [13], a large current signal is desirable meaning large tunneling conductance. However, higher order processes such as cotunneling [14,15] lead to a smearing of Coulomb blockade phenomena and a compromise must be found in practice. While the strong tunneling regime has been explored extensively by recent experiments [16–19], theoretical predictions remain limited. The theoretical work roughly splits into two groups. On the one hand, higher order perturbative results [20–24] were successful in explaining some of the recent experimental data, yet, the analysis typically is restricted to conductances at

most of order G_K . Based on the diagrammatic expansion, partial resummation techniques were used to obtain non-perturbative results [25–28], however, for a restricted set of charge states. The arbitrary cutoff necessary in these latter theories limits their use for direct comparison with experimental findings. Further progress can be made by using perturbative renormalization group techniques [29, 30]. Apart from these approaches based on diagrams generated by treating tunneling as a perturbation, a formally exact path integral expression [31] including all orders in the tunneling conductance may serve as a starting point for analytical predictions [32–36] and numerical calculations [37–39]. While perturbation theory in the tunneling term usually starts from states with definite electric charge, this latter approach employs the canonically conjugate phase variable and thus is well adapted to situations where the charge is smeared by thermal or quantum fluctuations. In this work we use the path integral approach to determine the linear conductance of single electron devices in the semiclassical limit. Some limiting cases of the results presented here were published in short form previously [40–42]. Here we give a fuller account of the approach and apply it to a larger variety of systems. New results are obtained for the frequency dependent conductance of a single tunnel junction embedded in an electromagnetic environment where we explicitly discuss the limit of a large tunnel conductance and low temperatures. Further, the approach has been extended to an array of junctions where we focus

^a e-mail: georg@tfp1.physik.uni-freiburg.de

on the dependence of the conductance on array length and environment.

The paper is organized as follows: in Section 2 we introduce the Hamiltonians of a tunnel junction and of the electromagnetic environment, respectively. We then explain the general method of calculating the linear conductance from the Kubo formula with the help of a generating functional. In Section 3 the case of a single tunnel junction embedded in an electromagnetic environment of arbitrary impedance is considered. We use this example to derive the effective action characterizing the generating functional which is employed also in subsequent sections with adequate generalizations. We evaluate the conductance in the semiclassical limit appropriate for high temperatures and/or large tunneling conductance and compare the results with experimental findings by Joyez *et al.* [17] and by Farhangfar *et al.* [18]. As a first extension of the method, we consider in Section 4 a linear array of tunnel junctions embedded in an electromagnetic environment. The conductance of the array is determined in the high temperature limit. Specifically, we study the effect of the environmental impedance on the conductance and show that with increasing length of the array the influence of the environment is strongly suppressed. In Section 5 we turn to a single electron transistor (SET). Here, we go beyond leading order in the semiclassical expansion and determine the conductance in dependence on the gate voltage. The findings are compared with experimental data by Joyez *et al.* [16] for SETs in the strong tunneling regime. We conclude and discuss possible extensions in Section 6.

2 Model and general method

In this section we introduce the Hamiltonian for a single tunnel junction and model the electromagnetic environment in terms of a set of LC circuits. A metal-oxide layer-metal tunnel junction consists of two metallic leads separated by a thin oxide layer [1, 43]. Provided the screening length in the metal is small compared to typical electrode and oxide barrier dimensions, one may introduce a geometrical capacitance C . The energy shift for an electron tunneling from one lead to the other is determined by the charging energy $E_C = e^2/2C$. The corresponding Coulomb Hamiltonian reads

$$H_C(Q) = \frac{Q^2}{2C}, \quad (1)$$

where Q is the charge operator on the capacitance. The leads are described by Fermi-liquid theory as free quasiparticles

$$H_{\text{qp}} = \sum_{k\sigma} \epsilon_{k\sigma} a_{k\sigma}^\dagger a_{k\sigma} + \sum_{q\sigma} \epsilon_{q\sigma} a_{q\sigma}^\dagger a_{q\sigma}, \quad (2)$$

where the $\epsilon_{p\sigma}$ are quasiparticle energies, and $a_{p\sigma}^\dagger$ and $a_{p\sigma}$ are creation and annihilation operators for states on the two electrodes, respectively. The indices $p = k, q$ are longitudinal quantum numbers and σ is the channel index including transversal and spin quantum numbers. Provided

the tunneling amplitudes are small, we may describe barrier transmission by a tunneling Hamiltonian [1, 44]

$$H_T(\varphi) = \sum_{kq\sigma} \left(t_{kq\sigma} a_{k\sigma}^\dagger a_{q\sigma} \Lambda + \text{H.c.} \right), \quad (3)$$

preserving the channel index σ . Here $t_{kq\sigma}$ is the tunneling amplitude and Λ the charge shift operator obeying $\Lambda^\dagger Q \Lambda = Q + e$. Defining a conjugate phase φ by $[Q, \varphi] = ie$, we may write

$$\Lambda = \exp(-i\varphi). \quad (4)$$

We mention that the conventional form of the tunneling Hamiltonian without an operator Λ , see *e.g.* [31], can be obtained *via* an unitary transformation. However, the form (3) is more convenient when Coulomb charging effects are relevant. The total Hamiltonian of a tunnel junction then reads

$$H_J(Q, \varphi) = H_C(Q) + H_{\text{qp}} + H_T(\varphi), \quad (5)$$

where the dependence on the charge and conjugate phase operators is made explicit to emphasize the similarity between the charging energy and a kinetic energy and between the tunneling Hamiltonian and an effective potential energy.

The electromagnetic environment can be described by a Caldeira-Leggett model [45] as a linear combination of LC circuits

$$H_{\text{em}}(\varphi) = \sum_{n=1}^N \left[\frac{Q_n^2}{2C_n} + \frac{1}{2L_n} \left(\frac{\hbar}{e} \right)^2 (\varphi - \phi_n)^2 \right], \quad (6)$$

coupled to the phase operator φ of the device. The parameters of the LC-circuits are related to the environmental admittance by

$$Y(\omega) = \sum_{n=1}^N \frac{\pi}{L_n} [\delta(\omega + \omega_n) + \delta(\omega - \omega_n)], \quad (7)$$

where the $\omega_n = 1/\sqrt{L_n C_n}$ are the eigenfrequencies of the oscillators. A single electron tunneling device consists of tunnel junctions, capacitances, and admittances. The Hamiltonian of this system can be constituted from the elements discussed above. Then, the bosonic degrees of freedom of the admittance and the fermionic degrees of freedom of the electrodes may be traced out. In the next section we exemplarily derive the Hamiltonian and the effective action for a tunnel junction embedded in an environment of arbitrary impedance.

Specifically, in this article we determine the linear conductance $G = \partial I / \partial V|_{V=0}$ of single electron devices. Here I is the measured current and V the applied voltage. The measured current I may be written as the expectation value of an operator $I^{(1)}$. Using linear response theory in the applied voltage V , one may deduce a Kubo like formula for the linear conductance

$$G(\omega) = \frac{1}{i\hbar\omega} \lim_{\nu_n \rightarrow \omega + i\delta} \int_0^{\hbar\beta} d\tau e^{i\nu_n \tau} \langle I^{(1)}(\tau) I^{(2)}(0) \rangle, \quad (8)$$

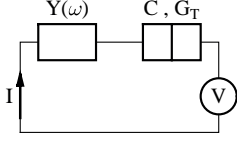


Fig. 1. Circuit diagram of a tunnel junction in series with an admittance.

where $I^{(2)}$ is a current operator determined by the coupling to the applied voltage V , see below. The $\nu_n = 2\pi n/\hbar\beta$ are Matsubara frequencies. Correlation functions can be written as variational derivatives

$$\langle I^{(1)}(\tau)I^{(2)}(\tau') \rangle = \frac{\hbar^2}{Z[0,0]} \frac{\delta^2 Z[\xi_1, \xi_2]}{\delta \xi_1(\tau) \delta \xi_2(\tau')} \Big|_{\xi_i=0} \quad (9)$$

of a generating functional [32]

$$Z[\xi_1, \xi_2] = \text{tr} T_\tau \exp \left\{ -\frac{1}{\hbar} \int_0^{\hbar\beta} d\tau \left[H - \sum_{i=1,2} I^{(i)} \xi_i(\tau) \right] \right\} \quad (10)$$

depending on auxiliary fields ξ_i . Here, the Hamiltonian H describes the system at vanishing external voltage, $V = 0$, and T_τ is the Matsubara time ordering operator. In subsequent sections we apply this generating functional approach to derive an explicit expression for the linear conductance in the semiclassical regime.

3 Tunnel junction with environment

3.1 Generating functional

We consider a tunnel junction characterized by the geometrical capacitance C and the tunneling conductance G_T embedded in an electromagnetic environment. *Via* network transformations it is always possible to transform the environmental degrees of freedom into an admittance $Y(\omega)$ in series with the junction biased by a voltage source V , *cf.* Figure 1. In this subsection we obtain the effective action characterizing the generating functional introduced above.

The Hamiltonian reads $H = H_J(Q_J, \varphi_J) + H_{\text{em}}(\varphi_{\text{em}})$. Here the phases φ_J and φ_{em} are related to the voltages V_J and V_{em} across the tunnel junction and the admittance, respectively, by $\dot{\varphi}_J = \frac{e}{\hbar} V_J$ and $\dot{\varphi}_{\text{em}} = \frac{e}{\hbar} V_{\text{em}}$. Further, one has to take care of constraints for the variables imposed by the circuit. Using Kirchhoff's law for the voltages, we find that the sum of the phases in the circuit loop in Figure 1 has to be constant, *i.e.* $\varphi_J + \varphi_{\text{em}} + \psi = \text{const.}$, where we have described the voltage source in terms of an additional phase [9]

$$\psi(t) = \frac{e}{\hbar} \int_{-\infty}^t dt' V(t'). \quad (11)$$

Similar relations hold for each loop of more complicated circuits. For an adequate handling of these constraints we

start from the Lagrangian description, $\mathcal{L} = T - U$. In general, the kinetic energy T is given by the sum of Coulomb energy terms and the effective potentials are the tunneling and environmental Hamiltonians. The constraints are naturally implemented by expressing the variables through generalized coordinates. Defining generalized momenta in the standard way, one can derive the Hamiltonian *via* a Legendre transformation. To define conjugate momenta non-ambiguously, we use the discrete Caldeira–Leggett model and perform the continuum limit only afterwards. Shunt capacitors need to be treated separately and will be discussed in Section 5. Since $\psi(t)$ is controlled externally, the phase φ_{em} may be eliminated in favor of $\varphi_J \equiv \varphi$ and we may write

$$H_{\text{JE}}(Q, \varphi) = H_J(Q, \varphi) + H_{\text{em}}(\varphi + \psi), \quad (12)$$

where $Q = \frac{e}{\hbar} \partial \mathcal{L} / \partial \dot{\varphi}$ is the momentum canonically conjugate to φ . In the second term, we have absorbed the minus sign in front of $\varphi + \psi$ into the arbitrary definition of the sign of the phase of the environment. The current may be defined as the time derivative of the charge on the geometrical capacitance

$$\dot{Q} = \frac{i}{\hbar} [H_{\text{JE}}, Q] = I_T + I_{\text{em}}, \quad (13)$$

where the charge can change either by discrete tunneling I_T across the capacitor or by continuous flow I_{em} in the leads. Here,

$$I_T = \frac{i}{\hbar} [H_T(\varphi), Q] = -\frac{ie}{\hbar} \sum_{kq\sigma} \left(t_{kq\sigma} a_{k\sigma}^\dagger a_{q\sigma} \Lambda - \text{H.c.} \right) \quad (14)$$

is the current through the tunnel junction and

$$I_{\text{em}}(\varphi) = \frac{i}{\hbar} [H_{\text{em}}(\varphi), Q] = \frac{\hbar}{e} \sum_{n=1}^N \frac{1}{L_n} (\varphi - \phi_n) \quad (15)$$

the current through the admittance at vanishing external voltage. To determine the linear conductance (8), we choose the measured current $I^{(1)}$ to be the current I_{em} , and $I^{(2)}$ follows from the coupling to the phase ψ in linear approximation: $H_{\text{JE}}(Q, \varphi, \delta\psi) = H_{\text{JE}}(Q, \varphi) + \frac{\hbar}{e} I^{(2)} \delta\psi$. *Via* a unitary transformation $U = \exp(-i\kappa_1 \psi Q/e)$, it is always possible to write the external voltage partly as a shift of the phase variable φ . Using $U^\dagger \varphi U = \varphi - \kappa_1 \psi$ and the general relation $H' = U^\dagger H U + i\hbar U^\dagger \frac{\partial}{\partial t} U$, we get

$$H'_{\text{JE}}(Q, \varphi) = H_J(Q + \kappa_1 V C, \varphi - \kappa_1 \psi) + H_{\text{em}}(\varphi + \kappa_2 \psi), \quad (16)$$

where κ_1 is an arbitrary shift and $\kappa_2 = 1 - \kappa_1$. Here we choose $\kappa_1 = 0$ so that the voltage couples solely to the environmental degrees of freedom and then get $H_{\text{em}}(\varphi + \delta\psi) = H_{\text{em}}(\varphi) + \frac{\hbar}{e} I_{\text{em}} \delta\psi$. Hence, in this case $I^{(2)}$ coincides with the measured current $I^{(2)} = I^{(1)} = I_{\text{em}}$. To derive the path integral representation of the generating

functional (10), we define

$$\tilde{H}_{\text{em}}(\varphi) = H_{\text{em}}(\varphi) - \xi(\tau)I_{\text{em}}(\varphi) = H_{\text{em}}[\varphi - \frac{e}{\hbar}\xi(\tau)] + \text{ind.} \quad (17)$$

where ind. denotes a φ -independent term that may be omitted. Further, we separate the exponential in equation (10) into a free part $A_0(\hbar\beta)$ for vanishing tunneling and a tunneling part $A_{\text{T}}(\hbar\beta)$ according to

$$T_{\tau}e^{-\frac{1}{\hbar}\int_0^{\hbar\beta} d\tau(\tilde{H}_0 + H_{\text{T}})} = A_0(\hbar\beta)A_{\text{T}}(\hbar\beta), \quad (18)$$

where

$$A_0(\tau) = T_{\tau}e^{-\frac{1}{\hbar}\int_0^{\tau} d\tau'\tilde{H}_0(\tau')} \quad (19)$$

describes the system in presence of the unperturbed Hamiltonian $\tilde{H}_0 = \tilde{H}_{\text{em}} + H_{\text{C}} + H_{\text{qp}}$. Using the series expansion of $A_{\text{T}}(\tau)$ in powers of H_{T} and separating the trace in equation (10) into partial traces over the charge degrees of freedom of the device, the quasiparticle components, and the environmental degrees of freedom, we obtain an expression of the generating functional as a sum of averages of the unperturbed system, *cf.* [20]. Due to the Coulomb interaction Hamiltonian H_{C} in the unperturbed Hamiltonian \tilde{H}_0 , contributions of a given order in H_{T} cannot simply be evaluated with the help of Wick's theorem, however, the partial traces over the quasiparticle components are averages weighted with the free fermionic density matrix $\sim \exp(-\beta H_{\text{qp}})$ and, accordingly, products of quasi-particle creation and annihilation operators in the interaction picture decompose into products of two-pair correlators. In the limit of large channel number $\mathcal{N} = \sum_{\sigma} 1 \gg 1$, only specific combinations of contractions contribute that may be written in terms of two-time correlators of the tunneling Hamiltonian

$$\begin{aligned} G(\tau, \tau') &= \frac{1}{\hbar^2} \langle H_{\text{T}}(\tau)H_{\text{T}}(\tau') \rangle_{\text{qp}} \\ &= \frac{t^2}{\hbar^2} \sum_{k_1 q_1 \sigma_1} \sum_{k_2 q_2 \sigma_2} \sum_{\zeta_1, \zeta_2 = \pm} \zeta_1 \zeta_2 A^{\zeta_1}(\tau) A^{-\zeta_2}(\tau') \\ &\quad \times \langle a_{k_1 \sigma_1}^{\zeta_1}(\tau) a_{k_2 \sigma_2}^{-\zeta_2}(\tau') \rangle_{\text{qp}} \langle a_{q_1 \sigma_1}^{-\zeta_1}(\tau) a_{q_2 \sigma_2}^{\zeta_2}(\tau') \rangle_{\text{qp}} \\ &= \frac{t^2}{\hbar^2} \sum_{k q \sigma \zeta} A^{\zeta}(\tau) A^{-\zeta}(\tau') \frac{e^{\zeta(\tau' - \tau)(\epsilon_{k\sigma} - \epsilon_{q\sigma})}}{(1 + e^{\zeta\beta\epsilon_{k\sigma}})(1 + e^{-\zeta\beta\epsilon_{q\sigma}})} \quad (20) \end{aligned}$$

with a real averaged tunneling matrix element $t = \overline{t_{kq\sigma}}$. Here $\langle \dots \rangle_{\text{qp}}$ denotes the thermal average over the quasiparticles with Hamiltonian H_{qp} . The time dependence in the interaction picture reads

$$H_{\text{T}}(\tau) = \exp\left(\frac{\tau}{\hbar}H_{\text{qp}}\right) H_{\text{T}} \exp\left(-\frac{\tau}{\hbar}H_{\text{qp}}\right) \quad (21)$$

and

$$A^{\zeta}(\tau) = A_0(-\tau)A^{\zeta}A_0(\tau). \quad (22)$$

Further, we have introduced the notation $a^+ = a^\dagger$, $a^- = a$, and $\Lambda^\pm = \exp(\mp i\varphi)$. Performing the continuum limit for the longitudinal quantum numbers k and q , we find

$$G(\tau, \tau') = \frac{1}{\hbar} \frac{G_{\text{T}}}{G_{\text{K}}} \alpha(\tau - \tau') [\Lambda^\dagger(\tau)\Lambda(\tau') + \Lambda(\tau)\Lambda^\dagger(\tau')] \quad (23)$$

where $G_{\text{T}}/G_{\text{K}} = 4\pi^2 t^2 \mathcal{N} \rho \rho'$ is the classical dimensionless tunneling conductance with the densities of states ρ and ρ' at the Fermi level in the left and right electrode, respectively. In our approach the limit of strong tunneling is defined by $\mathcal{N} \gg 1$, $t^2 \rho \rho' \ll 1$ such that $4\pi^2 t^2 \mathcal{N} \rho \rho' \gg 1$. Since for lithographically fabricated metallic tunnel junctions typically $\mathcal{N} \gtrsim 10^4$, $G_{\text{T}}/G_{\text{K}}$ can become very large, although each single channel is weakly transmitting only.

The quasiparticle excitations generated by H_{T} are described by an electron-hole pair Green function [20]

$$\begin{aligned} \alpha(\tau) &= \frac{1}{4\pi^2 \mathcal{N} \hbar \rho \rho'} \sum_{k q \sigma} \frac{e^{(\epsilon_{k\sigma} - \epsilon_{q\sigma})\tau}}{(1 + e^{-\beta\epsilon_{k\sigma}})(1 + e^{\beta\epsilon_{q\sigma}})} \\ &= \frac{\hbar}{4\pi^2} \int_{-\infty}^{\infty} d\epsilon \frac{\epsilon e^{-|\epsilon|/D}}{1 - e^{-\hbar\beta\epsilon}} e^{-\tau\epsilon} \quad (24) \end{aligned}$$

where the electron and hole propagate on different electrodes. D is the electronic bandwidth which may be set to infinity at the end of the calculation since $D \gg E_{\text{C}}, k_{\text{B}}T$. Due to analytic properties of thermal Green functions, we may write

$$\alpha(\tau) = \frac{1}{\hbar\beta} \sum_{n=-\infty}^{\infty} \tilde{\alpha}(\nu_n) e^{-i\nu_n \tau} \quad (25)$$

with Fourier coefficients

$$\tilde{\alpha}(\nu_n) = -\frac{\hbar}{4\pi} \overline{\nu_n} e^{-\overline{\nu_n}/D}. \quad (26)$$

Here and in the remainder of the article the analytic continuation of the absolute value is defined by

$$\overline{z} = \begin{cases} z & \text{Re}(z) > 0 \\ -z & \text{Re}(z) < 0, \end{cases} \quad (27)$$

which leads to a unique analytical continuation [46] of the Fourier coefficients (26). Along these lines the partial traces over the quasiparticle components may be evaluated in terms of the tunneling kernel $\alpha(\tau)$.

To proceed we need to consider next the partial trace over the charge degrees of freedom. It is convenient, to change to the phase representation and insert identity operators $\int d\varphi_{\tau} |\varphi_{\tau}\rangle \langle \varphi_{\tau}|$ at each imaginary time slice $\tau_n = \frac{\hbar\beta}{N}n$, $n = 0 \dots, N$ with $N \rightarrow \infty$. The charge shift operators in the interaction picture then become $\Lambda^\pm(\tau) = \exp(\mp i\varphi_{\tau})$. Dividing the generating functional (10) by the quasiparticle partition function $\text{tr}_{\text{qp}} \exp(-\beta H_{\text{qp}})$ which

has no effect on the correlator (9), we get

$$Z_{\text{JE}}[\xi] = \int \mathcal{D}[\varphi] \prod_{n=1}^N \int \mathcal{D}[\phi_n] \exp \left\{ -\frac{1}{\hbar} S_0[\varphi, \phi_n, \xi] \right\} \\ \times \sum_{m=0}^{\infty} \int_0^{\hbar\beta} d\tau_{2m} \int_0^{\tau_{2m}} d\tau_{2m-1} \dots \int_0^{\tau_2} d\tau_1 \\ \times \sum_{\text{pairs } k=1}^m \prod G(\tau_{k_1}, \tau_{k_2}), \quad (28)$$

where $S_0[\varphi, \phi_n, \xi] = S_{\text{C}}[\varphi] + S_{\text{em}}[\varphi, \phi_n, \xi]$ contains the environmental and the Coulomb actions specified below. Since the integrand is invariant under exchange of an arbitrary pair of variables we may extend the integrations to $\int_0^{\hbar\beta} d\tau_i$ ($i = 1, \dots, 2m$) and compensate the larger integration region by a factor $1/(2m)!$. Further the sum over pairs leads to a factor $(2m-1)!!$. Interchanging integrals and product we get

$$Z_{\text{JE}}[\xi] = \int \mathcal{D}[\varphi] \prod_{n=1}^N \int \mathcal{D}[\phi_n] \exp \left\{ -\frac{1}{\hbar} S_0[\varphi, \phi_n, \xi] \right\} \\ \times \sum_{m=0}^{\infty} \frac{1}{m!} \left[\frac{1}{2} \int_0^{\hbar\beta} d\tau \int_0^{\hbar\beta} d\tau' G(\tau, \tau') \right]^m \\ = \int \mathcal{D}[\varphi] \prod_{n=1}^N \int \mathcal{D}[\phi_n] \exp \left\{ -\frac{1}{\hbar} S_{\text{JE}}[\varphi, \phi_n, \xi] \right\}, \quad (29)$$

where the effective Euclidean action splits into three parts

$$S_{\text{JE}}[\varphi, \phi_n, \xi] = S_{\text{C}}[\varphi] + S_{\text{T}}[\varphi] + S_{\text{em}}[\varphi, \phi_n, \xi]. \quad (30)$$

Here

$$S_{\text{C}}[\varphi] = \int_0^{\hbar\beta} d\tau \frac{\hbar^2 C}{2e^2} \dot{\varphi}^2 \quad (31)$$

describes Coulomb charging and

$$S_{\text{T}}[\varphi] = 2 \frac{G_{\text{T}}}{G_{\text{K}}} \int_0^{\hbar\beta} d\tau \int_0^{\hbar\beta} d\tau' \alpha(\tau - \tau') \sin^2 \left[\frac{\varphi(\tau) - \varphi(\tau')}{2} \right] \quad (32)$$

quasi-particle tunneling across the junction. The environmental action is given by

$$S_{\text{em}}[\varphi, \phi_n, \xi] = \sum_{n=1}^N \int_0^{\hbar\beta} d\tau \left[\frac{\hbar^2 C_n}{2e^2} \dot{\phi}_n^2 + \frac{\hbar^2}{2e^2 L_n} \left(\varphi - \frac{e}{\hbar} \xi - \phi_n \right)^2 \right]. \quad (33)$$

The remaining trace over environmental degrees of freedom in equation (29) can be evaluated exactly [47] leading to a quadratic nonlocal action

$$S_{\text{Y}}[\varphi, \xi] = \frac{1}{2} \int_0^{\hbar\beta} d\tau \int_0^{\hbar\beta} d\tau' k(\tau - \tau') \left[\varphi(\tau) - \frac{e}{\hbar} \xi(\tau) - \varphi(\tau') + \frac{e}{\hbar} \xi(\tau') \right]^2, \quad (34)$$

where the kernel $k(\tau)$ can be written as a Fourier series (25) with coefficients

$$\tilde{k}(\nu_n) = -\frac{\hbar}{4\pi} \frac{\widehat{Y}(\overline{\nu}_n)}{G_{\text{K}} \nu_n}. \quad (35)$$

Here $\widehat{Y}(s)$ is the Laplace transform of the environmental response function $Y(t)$, *cf.* reference [47]. Due to causality, for $\text{Re}(s) > 0$, one may write $\widehat{Y}(s) = Y(is)$ where $Y(\omega)$ is the frequency dependent admittance (7) of the environment.

This way the generating functional reads

$$Z_{\text{JE}}[\xi] = \int \mathcal{D}[\varphi] \exp \left\{ -\frac{1}{\hbar} S_{\text{JE}}[\varphi, \xi] \right\}, \quad (36)$$

with the effective action

$$S_{\text{JE}}[\varphi, \xi] = S_{\text{C}}[\varphi] + S_{\text{T}}[\varphi] + S_{\text{Y}}[\varphi, \xi]. \quad (37)$$

The explicit form of the generating functional serves as a starting point to calculate the correlator in the next subsection.

3.2 Conductance

We now perform the functional derivatives in equation (9) explicitly and get for the correlator [40]

$$\langle I_{\text{em}}(\tau) I_{\text{em}}(0) \rangle = \frac{1}{Z_{\text{JE}}} \int \mathcal{D}[\varphi] \exp \left\{ -\frac{1}{\hbar} S_{\text{JE}}[\varphi, 0] \right\} \\ \times \left(2 \frac{e^2}{\hbar} k(\tau) + I_{\text{em}}[\varphi, \tau] I_{\text{em}}[\varphi, 0] \right), \quad (38)$$

where $Z_{\text{JE}} = Z_{\text{JE}}[0]$ denotes the partition function. The current functional $I_{\text{em}}[\varphi, \tau]$ arising as variational derivative of the effective action (37) reads

$$I_{\text{em}}[\varphi, \tau] = \frac{2e}{\hbar} \int_0^{\hbar\beta} d\tau' k(\tau - \tau') \varphi(\tau'). \quad (39)$$

The conductance (8) now splits into two pieces

$$G_{\text{JE}}(\omega) = G_{\text{JE}}^{(1)}(\omega) + G_{\text{JE}}^{(2)}(\omega), \quad (40)$$

where

$$G_{\text{JE}}^{(1)}(\omega) = \frac{1}{i\hbar\omega} \frac{2e^2}{\hbar} \tilde{k}(-i\omega + \delta) = Y(\omega) \quad (41)$$

corresponds to the first term in equation (38), and

$$G_{\text{JE}}^{(2)}(\omega) = \frac{1}{i\hbar\omega} F_{\text{JE}}(-i\omega + \delta) \quad (42)$$

with

$$F_{\text{JE}}(\nu_n) = \frac{1}{Z_{\text{JE}}} \int \mathcal{D}[\varphi] \exp \left\{ -\frac{1}{\hbar} S_{\text{JE}}[\varphi, 0] \right\} F[\varphi, \nu_n] \quad (43)$$

to the second term in equation (38). The explicit form of the auxiliary functional $F[\varphi, \nu_n]$ in terms of the Fourier components $\tilde{\varphi}(\nu_m)$ reads

$$F[\varphi, \nu_n] = \frac{4e^2\beta}{\hbar} \tilde{k}(\nu_n) \tilde{\varphi}(\nu_n) \sum_{m=-\infty}^{+\infty} \tilde{k}(\nu_m) \tilde{\varphi}(\nu_m). \quad (44)$$

So far no further approximations have been made and within the model used equations (40-44) give a formally exact representation of the linear conductance. To proceed we evaluate the path integral (43) in the semiclassical limit.

3.3 Semiclassical limit

The classical trajectory of the phase $\tilde{\varphi}$ is defined by $\delta S_{\text{JE}}[\tilde{\varphi}, 0]/\delta\tilde{\varphi} = 0$, and we obtain from equation (37) $\tilde{\varphi} = \varphi^0 = \text{const}$. Since the action is invariant under a global phase shift, we may put $\varphi^0 = 0$. Writing the action in terms of Fourier coefficients of the phase

$$\tilde{\varphi}(\nu_n) = \frac{1}{\hbar\beta} \int_0^{\hbar\beta} d\tau e^{i\nu_n\tau} \varphi(\tau) \quad (45)$$

and expanding in powers of $\tilde{\varphi}(\nu_n)$, we get

$$S_{\text{JE}}[\varphi] = S_{\text{JE}}^0[\varphi] + \sum_{k=2}^{\infty} S_{\text{JE}}^{2k}[\varphi], \quad (46)$$

where

$$S_{\text{JE}}^0[\varphi] = \hbar \sum_{n=1}^{\infty} \lambda_{\text{JE}}(\nu_n) |\tilde{\varphi}(\nu_n)|^2 \quad (47)$$

is the second order variational action with the eigenvalues

$$\lambda_{\text{JE}}(\nu_n) = \frac{\hbar^2\beta}{e^2} \overline{\nu_n} \left[\widehat{G}_0(\nu_n) + \widehat{Y}(\overline{\nu_n}) \right]. \quad (48)$$

Here

$$\widehat{G}_0(\nu_n) = \overline{\nu_n} C + G_{\text{T}} \quad (49)$$

describes the tunnel junction as a capacitance in parallel with an Ohmic resistor characterized by the classical tunneling conductance. Further

$$\begin{aligned} S_{\text{JE}}^{2k}[\varphi] &= \frac{G_{\text{T}}}{G_{\text{K}}} \frac{(-1)^{k+1}}{(2k)!} \sum_{l=1}^{2k-1} \binom{2k}{l} (-1)^l \hbar\beta \\ &\times \sum'_{n_1, \dots, n_{2k-1}} \tilde{\alpha} \left(-\sum_{p=1}^l \nu_{n_p} \right) \\ &\times \tilde{\varphi}(\nu_{n_1}) \cdots \tilde{\varphi}(\nu_{n_{2k-1}}) \tilde{\varphi} \left(-\sum_{p=1}^{2k-1} \nu_{n_p} \right) \end{aligned} \quad (50)$$

is the variational action of order $2k$. The summation over the n_i is over all integers with $n_i = 0$ omitted. Neglecting sixth and higher order terms, we get from equation (43)

$$\begin{aligned} F_{\text{JE}}(\nu_n) &= \frac{4e^2\beta}{\hbar} \frac{\tilde{k}(\nu_n)^2}{\lambda_{\text{JE}}(\nu_n)} \left[1 + \frac{G_{\text{T}}}{G_{\text{K}}} \frac{2\beta}{\lambda_{\text{JE}}(\nu_n)} \right. \\ &\times \left. \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \frac{\tilde{\alpha}(\nu_{n+m}) - \tilde{\alpha}(\nu_n) - \tilde{\alpha}(\nu_m)}{\lambda_{\text{JE}}(\nu_m)} \right]. \end{aligned} \quad (51)$$

The convergence of this expansion depends crucially on the eigenvalues (48). To estimate the range of validity of the truncated series, we write the smallest eigenvalue in more appropriate units as

$$\lambda_{\text{JE}}(\nu_1) = \frac{2\pi^2}{\beta E_{\text{C}}} + \frac{G_{\text{T}} + \widehat{Y}(\nu_1)}{G_{\text{K}}}. \quad (52)$$

This eigenvalue has to be large compared to 1, and we see that the expansion is useful for large conductances $G_{\text{T}} + \widehat{Y}(\nu_1) \gg G_{\text{K}}$ and/or high temperatures $\beta E_{\text{C}} \ll 2\pi^2$. Hence, we effectively expand in powers of

$$\epsilon = \text{Min} \left(\frac{G_{\text{K}}}{G_{\text{T}} + \widehat{Y}(\nu_1)}, \frac{\beta E_{\text{C}}}{2\pi^2} \right). \quad (53)$$

Performing the limit $i\nu_n \rightarrow \omega + i\delta$, the analytically continued eigenvalue (48) reads

$$\lambda_{\text{JE}}(-i\omega) = -i\omega \frac{\hbar^2\beta}{e^2} [G_0(\omega) + Y(\omega)], \quad (54)$$

where

$$G_0(\omega) = G_{\text{T}} - i\omega C \quad (55)$$

is the analytic continuation of the Laplace transform of equation (49). The relative minus sign of the capacitive term is due to the usual definition of the Fourier transform in quantum mechanics, the electro-technical convention is obtained by replacing $\omega \rightarrow -\omega$. For small frequencies the analytically continued eigenvalue (54) is no longer large compared to 1 and we are faced with a problem of order reduction. In the limit $i\nu_n \rightarrow \omega + i\delta$ each $1/\lambda_{\text{JE}}(\nu_n)$ in equation (51) becomes of order 1 while the $1/\lambda_{\text{JE}}(\nu_m)$ factors for $m \neq n$ are not analytically continued and remain of order ϵ . The correction term of order ϵ^2 in equation (51), that is the term proportional to $G_{\text{T}}/G_{\text{K}}$, becomes of order ϵ after analytic continuation. Hence we loose one factor of ϵ . Generally, one finds that the higher order variational actions (50) include at most one $1/\lambda_{\text{JE}}(\nu_n)$ factor and consequently are reduced at most by one order in ϵ . Thus, products of the form

$$S_{\text{JE}}^{2k_1}[\varphi] S_{\text{JE}}^{2k_2}[\varphi] \cdots S_{\text{JE}}^{2k_l}[\varphi] \quad (56)$$

of quartic or higher-order variational actions, as they arise from an expansion in powers of $\tilde{\varphi}(\nu_k)$, give quantum corrections of order $\epsilon^{k_1+k_2+\dots+k_l}$ and after analytical continuation of order $\epsilon^{k_1+k_2+\dots+k_l-1}$ and of higher orders.

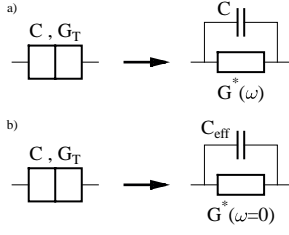


Fig. 2. Effective circuit diagrams for a tunnel junction in the semiclassical limit a) for arbitrary frequency and b) in the low frequency limit.

This proves that the terms of the expansion of F_{JE} given explicitly in equation (51) suffice to calculate the leading order quantum corrections. After performing the analytical continuation we get

$$G_{\text{JE}}^{(2)}(\omega) = -\frac{Y(\omega)^2}{G_0(\omega) + Y(\omega)} \left[1 + \frac{G_{\text{T}}}{G_0(\omega) + Y(\omega)} \mathcal{U}(\omega) \right] \quad (57)$$

with the quantum correction factor

$$\mathcal{U}(\omega) = \frac{2}{i\omega} \sum_{m=1}^{\infty} \nu_m \left[\frac{1}{\lambda_{\text{JE}}(\nu_m - i\omega)} - \frac{1}{\lambda_{\text{JE}}(\nu_m)} \right]. \quad (58)$$

Hence, for the total conductance we may write

$$G_{\text{JE}}(\omega) = \frac{G_{\text{eff}}(\omega)Y(\omega)}{G_{\text{eff}}(\omega) + Y(\omega)} \quad (59)$$

with an effective linear conductance of the junction

$$G_{\text{eff}}(\omega) = G_{\text{T}} [1 - \mathcal{U}(\omega)] - i\omega C. \quad (60)$$

This describes a linear element $G^*(\omega) = G_{\text{T}}[1 - \mathcal{U}(\omega)]$, depending on the whole circuit, in parallel with the geometrical junction capacitance C , cf. Figure 2a. The general form (59) is valid only to first order in ϵ . A systematic treatment of higher order contributions does not allow for a description of the tunnel junction in terms of an effective linear element. However, a partial resummation of higher order terms according to a self-consistent harmonic approximation [17, 48] leads again to the form (59).

3.4 Results and comparison with experimental data

For further discussion and comparison with experimental data we restrict ourselves to ohmic dissipation $Y(\omega) = Y$. The effective linear element (60) then reads

$$\frac{G^*(\omega)}{G_{\text{T}}} = 1 - \left[\frac{\psi(1+u+\tilde{\omega}) - \psi(1+\tilde{\omega})}{u} + \frac{\psi(1+u+\tilde{\omega}) - \psi(1+u)}{\tilde{\omega}} \right] \frac{\beta E_{\text{C}}}{\pi^2}, \quad (61)$$

where ψ is the logarithmic derivative of the gamma function and

$$u = g \frac{\beta E_{\text{C}}}{2\pi^2}, \quad \tilde{\omega} = \frac{\hbar\beta}{2\pi i} \omega \quad (62)$$

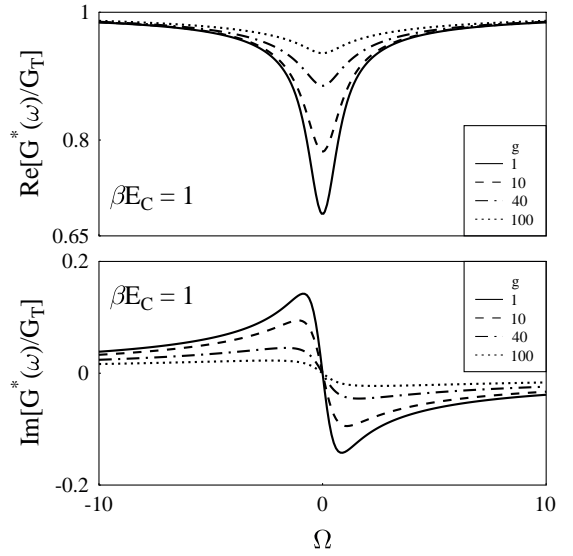


Fig. 3. Real and imaginary parts of $G^*(\omega)/G_{\text{T}}$ in the ohmic damping case for $\beta E_{\text{C}} = 1$ and various values of the dimensionless conductance g in dependence on the dimensionless frequency $\Omega = \hbar\omega/2\pi E_{\text{C}}$.

are auxiliary quantities. We also have introduced the dimensionless parallel conductance

$$g = (G_{\text{T}} + Y)/G_{\text{K}}. \quad (63)$$

The quantum corrections depend only on this combination of conductances. The real and imaginary parts of $G^*(\omega)/G_{\text{T}}$ are depicted in Figure 3 for $\beta E_{\text{C}} = 1$ and various values of g .

The quantum corrections are most pronounced at zero frequency and disappear nonalgebraically for large ω and/or u , due to the logarithmic behavior of the psi-function for large arguments.

For the dc conductance we get from (61)

$$\frac{G^*(\omega=0)}{G_{\text{T}}} = 1 - \left[\frac{\gamma + \psi(1+u)}{u} + \psi'(1+u) \right] \frac{\beta E_{\text{C}}}{\pi^2} \quad (64)$$

which coincides with our previous result [40]. In particular, in the limit of a very low resistance environment, the total conductance (59) approaches the classical limit nonanalytically, cf. Figure 4, leading to the asymptotic expansion [40]

$$G_{\text{JE}}(\omega=0) = G_{\text{T}} \left[1 + 2 \frac{G_{\text{K}}}{Y} \ln \left(\frac{G_{\text{K}}}{Y} \right) + \mathcal{O} \left(\frac{|\ln(\beta E_{\text{C}})| G_{\text{K}}}{Y} \right) \right]. \quad (65)$$

On the other hand for moderate to large environmental resistance, we may expand equation (64) with respect to u leading to a total conductance

$$G_{\text{JE}}(\omega=0) = \frac{G_{\text{T}}Y}{G_{\text{T}} + Y} \left\{ 1 - \frac{Y}{G + Y} \frac{\beta E_{\text{C}}}{3} + \mathcal{O} [(\beta E_{\text{C}})^2, u\beta E_{\text{C}}] \right\}. \quad (66)$$

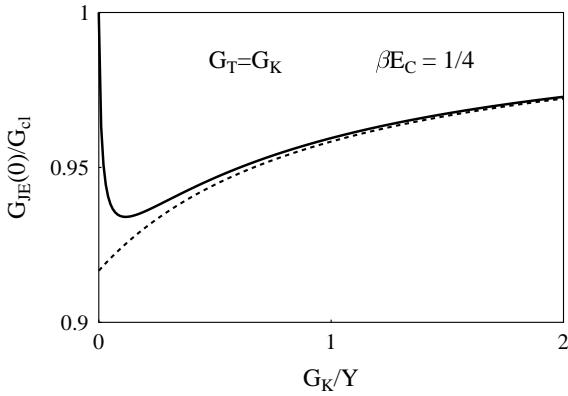


Fig. 4. The ratio of the total dc conductance $G_{JE}(0)$ and the classical conductance $G_{cl} = G_T Y / (G_T + Y)$ shown *vs.* the dimensionless environmental resistance G_K/Y for $G_T = G_K$ and $\beta E_C = 1/4$. The solid line is the result (59) for $\omega = 0$ and ohmic damping, and the dotted line corresponds to the approximation (66) for moderate to small Y .

This approximation corresponds to the dotted line in Figure 4 and remains analytic in the limit of large environmental conductance where it obviously fails.

In Figure 5 we compare our prediction (64) with recent experimental data by Joyez *et al.* [17] for dimensionless conductance $g = 4.2$ and 23.8 (upper plot) and by Farhangfar *et al.* [18] for $g = 4.52$ and 34.2 (lower plot). Figure 5 shows that in the limit of large conductance we are able to explain the whole range of temperatures explored experimentally, whereas for moderate conductance only the high temperature part is covered by the semiclassical theory. Here, the parameters g and E_C have not been adjusted to improve the fit but coincide with those given in the experimental papers.

We conclude this section with some remarks on the frequency dependence of the conductance that has not been studied experimentally, so far. For small frequencies we may expand the result (61) and write

$$G^*(\omega) = G^*(\omega = 0) - i\omega C^* + \mathcal{O}(\omega^2), \quad (67)$$

where C^* leads to a renormalization of the junction capacitance C . The renormalized capacitance $C_{\text{eff}} = C + C^*$ reads

$$\frac{C_{\text{eff}}}{C} = 1 + \frac{G_T}{G_K} \left[\frac{\frac{\pi^2}{3} - 2\psi'(1+u)}{u} - \psi''(1+u) \right] \frac{(\beta E_C)^2}{4\pi^4}. \quad (68)$$

The correction shows a quadratic dependence on βE_C and therefore is suppressed at high temperatures. It also vanishes linearly for large conductance g due to the analytical properties of the psi function. The semiclassical treatment covers only the region of weak Coulomb blockade. Whereas for small tunneling conductance low temperatures imply strong Coulomb blockade, these effects are suppressed for highly conducting tunnel junctions and the semiclassical theory is restored. A closer examination of equation (61)

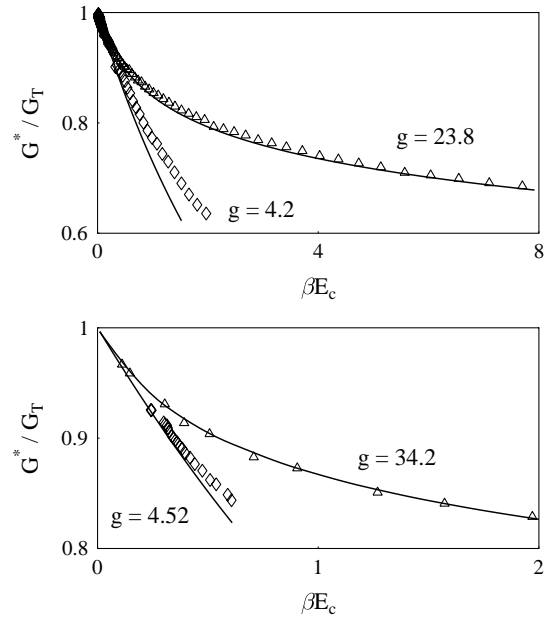


Fig. 5. The renormalized conductance (64) *versus* the dimensionless temperature compared with experimental data by Joyez *et al.* [17] for dimensionless parallel conductance $g = 4.2$ and 23.8 (upper plot) and with experimental data by Farhangfar *et al.* [18] for $g = 4.52$ and 34.2 (lower plot).

shows that for $g \gg 2 \ln(\beta E_C)$ the quantum corrections are always small. For fixed $g \gg 1$, our predictions are therefore valid for a very large range of temperatures covering in fact the entire range of parameters presently attainable experimentally for metallic junctions with strong tunneling [17–19]. Figure 6 depicts the real and imaginary parts of $G^*(\omega)/G_T$ for $g = 60$ and various temperatures. With decreasing temperature the real part shows for $\omega = 0$ a logarithmic decrease, $G^*/G_T = 1 - 2 \ln(\beta E_C)/g$, as long as $k_B T \gg E_C \exp(-g/2)$. Thus, for large conductance the semiclassical treatment is an effective high temperature expansion valid for $k_B T$ large compared with the renormalized charging energy $E_C^* \approx E_C \exp(-g/2)$ [33,36]. Moreover, the analytical form of the quantum corrections indicates that Coulomb blockade survives for arbitrary large conductance but becomes strong only for temperatures below E_C^*/k_B .

In the limit $T \rightarrow 0$ and $g \rightarrow \infty$ such that $k_B T \gg E_C \exp(-g/2)$, the imaginary part of $G^*(\omega)$ becomes a step function of width $2\pi/g$, *cf.* Figure 6, leading to a divergent renormalized capacitance of the form $C_{\text{eff}}/C = \beta E_C G_T / 6(G_T + Y)$. The linear dependence of C_{eff} on β starts already at very high temperatures, *cf.* Figure 7, and only saturates for βE_C of order $\exp(g/2)$. The large renormalized capacitance is a strong tunneling effect due to multiple electron tunneling and is found likewise for non-Ohmic environmental impedances. In Figure 7 we show the renormalized capacitance C_{eff}/C for $G_T/G_K = 20$ and various values of Y/G_K in dependence on the dimensionless inverse temperature βE_C . Note that the linear behavior of the capacitance starts already near $\beta E_C = 1$.

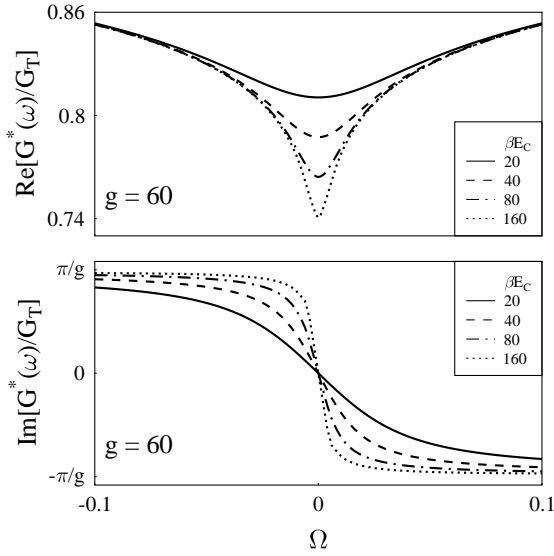


Fig. 6. Real and imaginary parts of $G^*(\omega)/G_T$ in the Ohmic damping case for dimensionless conductance $g = 60$ and inverse temperatures $\beta E_C = 20, 40, 80,$ and 160 in dependence on the dimensionless frequency $\Omega = \omega\hbar/2\pi E_C$.

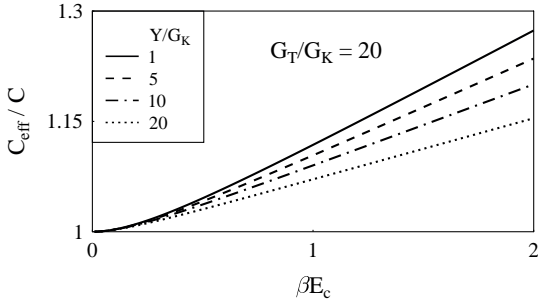


Fig. 7. Renormalized capacitance C_{eff}/C in the Ohmic damping case for tunneling conductance $G_T/G_K = 20$ and various environmental conductances $Y/G_K = 1, 5, 10,$ and 20 in dependence on the dimensionless inverse temperature βE_C .

The renormalized capacitance describes the frequency dependence of the conductance for small frequencies $\omega C_{\text{eff}} \ll \pi G_T/g$, cf. Figure 6. Rewriting this inequality we get $\omega \ll 6k_B T/\hbar \approx 10^{11} T$ Hz, where T is the temperature measured in Kelvin. Thus for all accessible temperatures the frequency range of strong $1/f$ noise can be avoided, and the effect predicted should be clearly observable experimentally.

4 Array of junctions with environment

4.1 Generating functional and conductance

As a first extension of the method, we now consider linear arrays of N tunnel junctions embedded in an electromagnetic environment. The junctions are characterized by classical tunneling conductances G_j and geometrical capacitances C_j in parallel. Like in the previous section, the

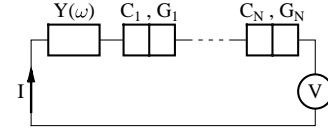


Fig. 8. Circuit diagram of an array of N tunnel junctions in series with an admittance $Y(\omega)$.

environment can be transformed into an admittance $Y(\omega)$ in series with an array of junctions biased by a voltage source V , cf. Figure 8. We start with a Lagrangian description depending on phase variables φ_j of each junction $j = 1 \dots N$ and an environmental phase φ_{em} with the constraint $\sum_{j=1}^N \varphi_j + \varphi_{\text{em}} + \psi = \text{const.}$, where ψ describes the applied voltage and is given by equation (11). Using the φ_j , $j = 1 \dots N$ as generalized variables we find for the total Hamiltonian

$$H_{\text{AE}}(\{Q_j\}, \{\varphi_j\}) = \sum_{j=1}^N H_J(Q_j, \varphi_j) + H_{\text{em}} \left(\sum_{j=1}^N \varphi_j + \psi \right) \quad (69)$$

with the junction and environmental Hamiltonians defined by equations (1-6). We follow the analysis in the previous section and first derive a formally exact expression for the linear conductance. As measured current $I^{(1)}$ we choose again the current flowing through the environmental impedance given by equation (15) with φ replaced by $\sum_{j=1}^N \varphi_j$ and Q by $\sum_{j=1}^N Q_j$, respectively. The second current operator $I^{(2)}$ is determined by the linear coupling to ψ and we get $I^{(1)} = I^{(2)} = I_{\text{em}}$. Following the lines of reasoning in the previous sections, the generating functional is found to read

$$Z_{\text{AE}}[\xi] = \int D[\{\varphi_j\}] \exp \left\{ -\frac{1}{\hbar} S_{\text{AE}}[\{\varphi_j\}, \xi] \right\}, \quad (70)$$

with the effective Euclidean action

$$S_{\text{AE}}[\{\varphi_j\}, \xi] = S_Y \left[\sum_{i=j}^N \varphi_j, \xi \right] + \sum_{i=j}^N S_j[\varphi_j]. \quad (71)$$

Here S_Y was introduced in equation (34) and $S_j[\varphi_j] = S_j^C[\varphi_j] + S_j^T[\varphi_j]$ describes the j th junction where the Coulomb action S_j^C and the tunneling action S_j^T are given by equations (31, 32), with the replacements $G_T \rightarrow G_j$ and $C \rightarrow C_j$. Performing the functional derivatives explicitly, the current-current correlator is found to be of the form (38) with the replacement $Z_{\text{JE}} \rightarrow Z_{\text{AE}} = Z_{\text{AE}}[0]$ and the appropriate action $S_{\text{AE}}[\{\varphi_j\}, 0]$. Further, the current functionals $I_{\text{em}}[\varphi, \tau]$ now depend on the sum of phases, $\varphi \rightarrow \sum_{j=1}^N \varphi_j$, and the functional integral is defined over all configurations of the phases φ_j . As in equation (41) the first term can be handled exactly, and we find $G_{\text{AE}}(\omega) = Y(\omega) + G_{\text{AE}}^{(2)}(\omega)$, where $G_{\text{AE}}^{(2)}(\omega)$ given by equations (42, 43) with $F[\varphi, \nu_n] \rightarrow F[\sum_{j=1}^N \varphi_j, \nu_n]$. So far no approximations

have been made and the result follows from a straightforward extension of our findings for a single junction. The qualitative difference lies in the topological structure of the phase configuration space and becomes clear when one evaluates the path integral. Again, we restrict ourselves to the semiclassical limit.

4.2 Semiclassical limit

Determining the classical path one has to take into account the topological structure of the configuration space. The phases φ_j , $j = 1, \dots, N$ are canonically conjugate to the charges Q_j on the junction capacitances. Now the array has $N - 1$ metallic islands in between the junctions carrying the charges $q_j = Q_j - Q_{j+1}$ for $j = 1, \dots, N - 1$. These island charges are quantized in units of the elementary charge e , and the phases ψ_j canonically conjugate to the q_j are compact, *i.e.*, the configuration space of the phases is a $(N - 1)$ -dimensional torus. Accordingly, the path integral is over all configurations of the phases ψ_j with $\psi_j(\hbar\beta) = \psi_j(0) + 2\pi k_j$ where the winding numbers k_j are integers. On the other hand, the environmental phase is extended and conjugate to a certain linear combination Q of the Q_j . Since the environment transfers charges continuously, Q is not quantized and the path integral over the environmental phase is over all configurations with $\varphi_{\text{em}}(0) = \varphi_{\text{em}}(\hbar\beta)$. Rather than making the canonical transform to the charges q_j ($j = 1, \dots, N - 1$), Q and the conjugate phases explicitly, one finds that equivalently we may integrate over all configurations of the phases φ_j with $\varphi_j(\hbar\beta) = \varphi_j(0) + 2\pi k_j$ where the integer winding numbers obey the constraint $\sum_{j=1}^N k_j = 0$.

At high temperatures the classical paths are straight line flips $\check{\varphi}_j^{(k_j)}(\tau) = \varphi_j^0 + \nu_{k_j}\tau$ running from φ_j^0 to $\varphi_j^0 + 2\pi k_j$. The action is invariant under global shifts of the φ_j and we may set $\varphi_j^0 = 0$ for all $j = 1, \dots, N$. All paths with winding number $k_j \neq 0$ are exponentially suppressed by the classical action contribution $S_j^{\text{cl}} \approx \pi^2 k_j^2 / \beta E_{C_j} + |k_j| G_j / 2G_K$. Thus, to obtain the leading order quantum corrections, we may restrict ourselves to winding numbers $k_j = 0$ for $j = 1 \dots N$. Finite winding numbers are considered in the next section where we focus on the single electron transistor and go beyond the leading order quantum correction.

The action may be expanded in powers of the Fourier coefficients $\tilde{\varphi}_i(\nu_n)$ yielding a result of the form (46) where the second order variational action reads

$$S_{\text{AE}}^0[\{\varphi_j\}] = S_Y^0 \left[\sum_{j=1}^N \varphi_j \right] + \sum_{j=1}^N S_j^0[\varphi_j]. \quad (72)$$

The environmental contribution is given by

$$S_Y^0[\varphi] = \hbar \sum_{n=1}^{\infty} \lambda_Y(\nu_n) |\tilde{\varphi}(\nu_n)|^2, \quad (73)$$

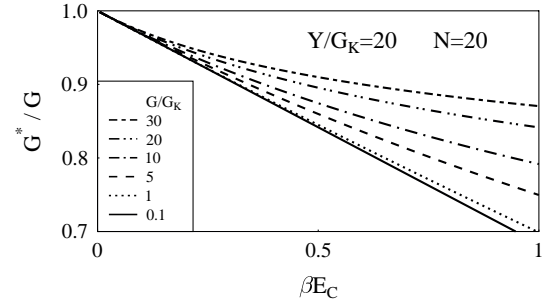


Fig. 9. Renormalized conductance G^*/G of an array of $N = 20$ tunnel junctions in dependence on βE_C for $Y/G_K = 20$ and various tunneling conductances G/G_K .

with the eigenvalues

$$\lambda_Y(\nu_n) = \frac{\hbar^2 \beta}{e^2} \nu_n \hat{Y}(\nu_n). \quad (74)$$

The second order variational tunneling action for junction j reads

$$S_j^0[\varphi_j] = \hbar \sum_{n=1}^{\infty} \lambda_j(\nu_n) |\tilde{\varphi}_j(\nu_n)|^2 \quad (75)$$

with the eigenvalues

$$\lambda_j(\nu_n) = \frac{\hbar^2 \beta}{e^2} \nu_n \hat{G}_j^0(\nu_n), \quad (76)$$

where $\hat{G}_j^0(\nu_n)$ is given by equation (49) adapted to a junction with capacitance C_j in parallel with an Ohmic resistor $1/G_j$. The higher order variational actions are given by straightforward extensions of equation (50). Expanding in powers of the higher order terms $S_{\text{AE}}^{2k}[\varphi]$, $k = 2, 3, \dots$, we are left with expectation values of products of the phase variables $\tilde{\varphi}_j(\nu_n)$. It is now useful to define a Gaussian average

$$\langle X \rangle_0 = \frac{1}{Z_{\text{AE}}^0} \int \prod_{n=1}^{\infty} \prod_{j=1}^N d\tilde{\varphi}_j(\nu_n) d\tilde{\varphi}_j^*(\nu_n) \times \exp \left\{ -\frac{1}{\hbar} S_{\text{AE}}^0[\{\varphi_j\}] \right\} X \quad (77)$$

with the Gaussian partition function Z_{AE}^0 defined by the requirement $\langle 1 \rangle_0 = 1$. The difference between Z_{AE}^0 and the full partition function Z_{AE} is of order $(\beta E_{C_j})^2$ and may be neglected here. Due to the Gaussian form of the measure $\exp[-S_{\text{AE}}^0[\{\varphi_j\}]/\hbar]$, the averages of products of Fourier coefficients $\tilde{\varphi}_j(\nu_n)$ decompose into sums over products of two point expectations. For two different phase variables $l \neq l'$ we obtain

$$\langle \tilde{\varphi}_l(\nu_n) \tilde{\varphi}_{l'}(\nu_m) \rangle_0 = -\delta_{n,-m} \prod_{j \neq l, l'}^{N+1} \lambda_j(\nu_n) / \Lambda(\nu_n), \quad (78)$$

with

$$A(\nu_n) = \sum_{i=1}^{N+1} \prod_{j \neq i}^{N+1} \lambda_j(\nu_n). \quad (79)$$

Here and in the remainder we define $\lambda_{N+1}(\nu_n) = \lambda_Y(\nu_n)$ and note that summation and multiplication indices run from 1 if not otherwise specified. For phase variables of the same junction we find

$$\langle \tilde{\varphi}_l(\nu_n) \tilde{\varphi}_l(\nu_m) \rangle_0 = \frac{1}{\lambda^{(l)}(\nu_n)} \delta_{n,-m}, \quad (80)$$

where

$$\lambda^{(l)}(\nu_n) = \frac{\hbar^2 \beta}{e^2} \frac{\hat{G}'_l(\nu_n)}{\nu_n} \quad (81)$$

plays the role of an effective eigenvalue for phase fluctuations in junction l with all other phases φ_j , $j \neq l$ already traced out. Here,

$$\hat{G}'_l(\nu_n) = \hat{G}_l^0(\nu_n) + \left[\frac{1}{\hat{Y}(\bar{\nu}_n)} + \sum_{j \neq l}^N \frac{1}{\hat{G}_j^0(\nu_n)} \right]^{-1} \quad (82)$$

may be considered as the Laplace transform of an effective response function describing the circuit seen from junction l , *i.e.*, a series of $N - 1$ junctions and an environmental impedance in parallel to junction l where the junctions are described effectively by linear elements. Including the fourth order variational derivative of the action, we get as a generalization of equation (51)

$$\begin{aligned} F_{\text{AE}}(\nu_n) &= \frac{4e^2 \beta \tilde{k}(\nu_n)^2}{\hbar} \frac{1}{A(\nu_n)} \sum_{l=1}^N \prod_{j \neq l}^N \lambda_j(\nu_n) \\ &\times \left[1 + \frac{G_l}{G_K} \frac{2\beta}{A(\nu_n)} \prod_{j \neq l}^N \lambda_j(\nu_n) \right. \\ &\times \left. \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \frac{\tilde{\alpha}(\nu_{n+m}) - \tilde{\alpha}(\nu_n) - \tilde{\alpha}(\nu_m)}{\lambda^{(l)}(\nu_m)} \right]. \quad (83) \end{aligned}$$

Now, the convergence of the expansion depends on the effective eigenvalues (81). To estimate the range of validity, we consider the smallest eigenvalues which at high temperatures are given by $\lambda^{(l)}(\nu_1) \approx 2\pi^2/\beta E_{C_l}$. Again the analytic continuation gives rise to a reduction of the order of the quantum corrections in the expansion parameter. For contributions with vanishing winding number the arguments given in the previous section apply likewise to the present problem. On the other hand, for lower temperatures one has to take into account winding numbers $k_j \neq 0$ and finds that some of the eigenvalues tend to zero. The marginally stable fluctuation modes lead to a breakdown of the simple semiclassical treatment. The appropriate extension of the semiclassical approximation was discussed elsewhere [36]. The topological structure of the phase space leading here to a breakdown of the simple

semiclassical approximation at low temperatures even for large conductance is the main difference between the single tunnel junction with environment and circuits containing many junctions. As a result one finds that the truncated expression (83) is valid up to first order in $\epsilon = \text{Max}(\beta E_{C_j} : j = 1, \dots, N)$. After the analytical continuation $\nu_n \rightarrow -i\omega + \delta$ we can write the total conductance in the compact form

$$G_{\text{AE}}(\omega) = \left[\frac{1}{Y(\omega)} + \sum_{j=1}^N \frac{1}{G_{\text{eff}}^j(\omega)} \right]^{-1}, \quad (84)$$

describing $N + 1$ linear elements in series: $G_{\text{eff}}^j(\omega)$ with $j = 1, \dots, N$ and the admittance $Y(\omega)$. Here, the $G_{\text{eff}}^j(\omega)$ are of the form (60) where the auxiliary functions \mathcal{U}_j are given by equation (58) with λ_{JE} replaced by $\lambda^{(j)}$ introduced in equation (81). This is a straightforward extension of the result in the previous section valid to linear order in ϵ for arbitrary conductances G_j and admittances $Y(\omega)$.

4.3 Discussion of results

For a more explicit discussion of the results we consider now N identical junctions $G_j = G$ and $C_j = C$. The eigenvalues (81) then read

$$\begin{aligned} \lambda(\nu_n) &= \lambda^{(j)}(\nu_n) \\ &= \frac{\hbar^2 \beta}{e^2} \frac{\hat{G}'_j(\nu_n)}{\nu_n} \left[\hat{G}_j^0(\nu_n) + \frac{\hat{Y}(\bar{\nu}_n) \hat{G}_j^0(\nu_n)}{(N-1)\hat{Y}(\bar{\nu}_n) + \hat{G}_j^0(\nu_n)} \right] \quad (85) \end{aligned}$$

and coincide for all junctions. For the total conductance of the array (84) we obtain

$$G(\omega) = \frac{G_{\text{eff}}(\omega) Y(\omega)}{G_{\text{eff}}(\omega) + N Y(\omega)}, \quad (86)$$

where

$$G_{\text{eff}}(\omega) = G[1 - \mathcal{U}(\omega)] - i\omega C \quad (87)$$

is the effective linear conductance of one junction. In this order each junction can be described by a linear element $G^*(\omega) = G[1 - \mathcal{U}(\omega)]$, depending on the circuit, in parallel with the geometrical capacitance C as depicted in Figure 2a. To proceed we consider an Ohmic environment $Y(\omega) = Y$ and find for the effective linear element

$$\begin{aligned} \frac{G^*(\omega)}{G} &= 1 - \left\{ \frac{(N-1)[\psi(1+u_T + \tilde{\omega}) - \psi(1+\tilde{\omega})]}{u_T} \right. \\ &+ \frac{\psi(1+u_N + \tilde{\omega}) - \psi(1+\tilde{\omega})}{u_N} \\ &+ \frac{(N-1)[\psi(1+u_T + \tilde{\omega}) - \psi(1+u_T)]}{\tilde{\omega}} \\ &+ \left. \frac{\psi(1+u_N + \tilde{\omega}) - \psi(1+u_N)}{\tilde{\omega}} \right\} \frac{\beta E_C}{\pi^2 N}, \quad (88) \end{aligned}$$

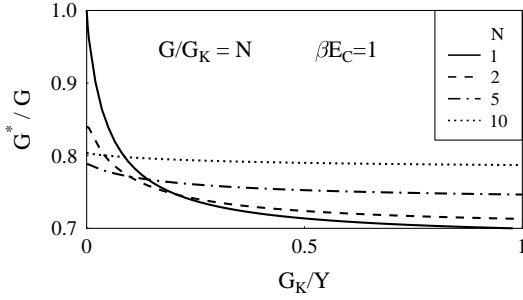


Fig. 10. Renormalized conductance G^*/G of an array of $N = 1, 2, 5,$ and 10 equivalent tunnel junctions leading to the same classical series conductance for $\beta E_C = 1$ as a function of the inverse environmental conductance G_K/Y .

where

$$u_N = \frac{G + NY}{G_K} \frac{\beta E_C}{2\pi^2}, \quad u_T = \frac{G}{G_K} \frac{\beta E_C}{2\pi^2}, \quad \tilde{\omega} = \frac{\hbar\beta}{2\pi} \omega \quad (89)$$

are auxiliary quantities and $E_C = e^2/2C$ is the charging energy for *one* junction. For $N = 1$ we recover the results of Section 3, of course. For a large array with $N \gg 1$, terms in equation (88) containing u_N drop out, and the quantum suppression becomes independent of Y . Furthermore the high-temperature anomaly, *cf.* Figure 4, is now a $1/N$ effect and the limiting result for $N \rightarrow \infty$ is analytic.

For small frequencies the effective element behaves like an Ohmic resistor $1/G^*(\omega = 0)$ with a renormalized capacitance in parallel. The dc conductance is given by

$$\frac{G^*(\omega = 0)}{G} = 1 - \left\{ (N-1) \left[\frac{\gamma + \psi(1+u_T)}{u_T} + \psi'(1+u_T) \right] + \frac{\gamma + \psi(1+u_N)}{u_N} + \psi'(1+u_N) \right\} \frac{\beta E_C}{N\pi^2}. \quad (90)$$

For $N = 2$ and $Y/G_K \rightarrow \infty$ this reduces to

$$\frac{G^*}{G} = 1 - \left[\frac{\gamma + \psi(1+u_T)}{u_T} + \psi'(1+u_T) \right] \frac{\beta E_C/2}{\pi^2} \quad (91)$$

coinciding with earlier findings for the symmetrical SET [35,41]. On the other hand, in the limit of large N and moderate $G \lesssim G_K$, equation (90) reduces to

$$\frac{G^*}{G} = 1 - \frac{N-1}{N} \frac{\beta E_C}{3} \quad (92)$$

again in accordance with earlier findings [49] derived from rate theory for small tunneling conductances. To discuss the strong tunneling corrections, we show in Figure 9 the renormalized conductance of an array of $N = 20$ tunnel junctions in dependence on βE_C for $Y/G_K = 20$ and various tunneling conductances G/G_K . We find that the weakly conducting case $G/G_K = 0.1$ perfectly coincides with the limiting formula (92) (both depicted by the solid

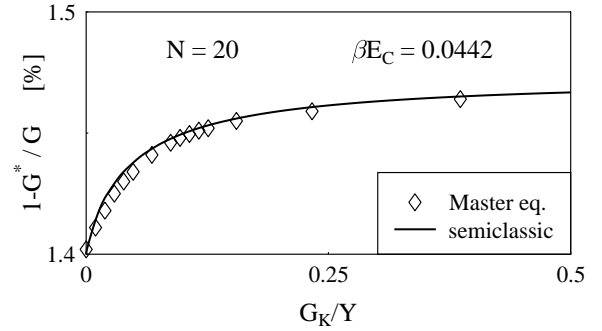


Fig. 11. Zero bias dip $1 - G^*/G_T$ in per cent for an array of length $N = 20$ and $\beta E_C = 0.0442$ as a function of the inverse environmental conductance G_K/Y in the perturbative limit compared with a numerical master equation approach by Farhangfar *et al.* [50].

line) whereas for larger tunneling conductances strong deviations from this behavior appear.

Experimentally one is interested in the dependence on the array length at fixed classical series conductance. In Figure 10 we show G^*/G for various N whereby G increases with the array length to keep the total classical series conductance constant. Whereas for $N < 5$ the renormalized conductance depends strongly on Y , it becomes independent for large N .

The comparison with available experimental data [50] is complicated by the large number of parameters, in particular, the charging energy differs from sample to sample with different array length. To test our predictions at least in the perturbative regime, we compare with the results of a master equation approach [50] based on the $P(E)$ theory [9]. In Figure 11 we show the zero bias dip $1 - G^*/G_T$ in per cent for an array of length $N = 20$ and $\beta E_C = 0.0442$ in the limit $G \rightarrow 0$. We find good agreement between the numerical calculations by Farhangfar *et al.* [50] and the analytical semiclassical result (90).

The renormalized capacitance C_{eff} includes the linear part in ω of $G^*(\omega)$ and the geometrical capacitance C and reads

$$\frac{C_{\text{eff}}}{C} = 1 + \frac{G}{G_K} \times \left\{ (N-1) \left[\frac{\frac{\pi^2}{3} - 2\psi'(1+u_T)}{u_T} - \psi''(1+u_T) \right] + \frac{\frac{\pi^2}{3} - 2\psi'(1+u_N)}{u_N} - \psi''(1+u_N) \right\} \frac{(\beta E_C)^2}{4N\pi^4} \quad (93)$$

showing a quadratic dependence on βE_C . The renormalization is suppressed at high temperatures and also vanishes linearly for large conductance in accordance with the behavior of a single tunnel junction with environment.

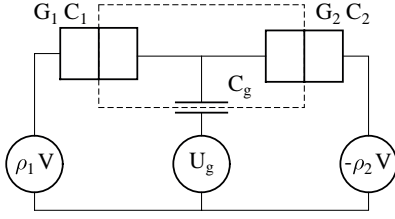


Fig. 12. Circuit diagram of the single electron transistor.

5 Single electron transistor

5.1 Generating functional and conductance

The SET consists of two tunnel junctions with tunneling conductances G_1 , G_2 and capacitances C_1 , C_2 , respectively, biased by a voltage source V , *cf.* Figure 12. The voltage may be split among the branches in $\rho_1 V$ and $\rho_2 V$ with $(\rho_1 + \rho_2 = 1)$. The island in between the junctions is connected via a gate capacitance C_g to a control voltage U_g shifting the electrostatic energy of the system continuously. The important energy scale is the charging energy $E_C = e^2/2C$ with the island capacitance $C = C_1 + C_2 + C_g$. For weak electron tunneling, E_C is the energy needed to charge the island with one excess electron at vanishing gate voltage $U_g = 0$. Due to the periodicity of the Hamiltonian in U_g , the conductance is a periodic function with period 1 of the dimensionless gate voltage $n_g = U_g C_g / e$ [7]. Following the lines of reasoning in the previous sections, we start with the Lagrangian description with phases φ_1 , φ_2 across the tunnel junctions and φ_g across the gate capacitor. Here, we treat the shunt capacitor C_g explicitly and do not introduce an effective environmental impedance. For the circuit depicted in Figure 12 there are two independent circuit loops leading to the constraints $\varphi_1 - \varphi_2 - \psi = \text{const.}$ and $\varphi_1 - \varphi_g - \psi_g - \rho_1 \psi = \text{const.}$, where ψ is the phase (11) of the transport voltage V and ψ_g the corresponding phase of the gate voltage U_g . Eliminating φ_2 and φ_g in favor of $\varphi \equiv \varphi_1$, we find

$$H'_{\text{SET}} = H_C(Q + en_g + V(C_2 + \rho_1 C_g)) + H_T^{(1)}(\varphi) + H_T^{(2)}(\varphi - \psi) + H_{\text{qp}}^{(1)} + H_{\text{qp}}^{(2)}. \quad (94)$$

The Coulomb Hamiltonian H_C is given by equation (1) and the other terms are defined by equations (2, 3) with corresponding labels. After a unitary transformation one finds equivalently

$$H_{\text{SET}} = H_C(Q + e\tilde{n}_g) + H_T^{(1)}(\varphi + \kappa_1 \psi) + H_T^{(2)}(\varphi - \kappa_2 \psi) + H_{\text{qp}}^{(1)} + H_{\text{qp}}^{(2)} \quad (95)$$

with an arbitrary shift parameter κ_1 and $\kappa_2 = 1 - \kappa_1$. Here we introduced a shifted dimensionless gate voltage

$$e\tilde{n}_g = U_g C_g + V(C_2 + \rho_1 C_g - \kappa_1 C). \quad (96)$$

In the sequel we restrict ourselves to zero frequency where the expectation values of the current operators through

both junctions coincide $\langle I_1 \rangle(\omega = 0) = \langle I_2 \rangle(\omega = 0)$. The first current operator $I^{(1)}$ may then be chosen as an arbitrary linear combination $I^{(1)} = \epsilon_1 I_1 - \epsilon_2 I_2$ (with $\epsilon_1 + \epsilon_2 = 1$) of the tunneling current operators I_1 and I_2 through junctions 1 and 2, respectively. The relative minus sign comes from the opposite directions of I_1 and I_2 , which are both positive for flux onto the island. The second current operator $I^{(2)} = \kappa_1 I_1 - \kappa_2 I_2$ is determined as above by the linear coupling term to the transport voltage V . The dependence of the Coulomb Hamiltonian on the transport voltage may be removed by a gate voltage shift and thus need not be considered. Moreover, this coupling would lead to a displacement current contribution vanishing at $\omega = 0$. To evaluate the current-current correlator we employ the generating functional (10). The current operators through the junctions are given by equation (14), for $j = 1, 2$. To derive the generating functional we define

$$\tilde{H}_T^{(j)} = H_T^{(j)}(\varphi) - I_j \xi_j(\tau). \quad (97)$$

The new tunneling Hamiltonians are of the form (3) where Λ is replaced by $[1 + ie\xi_j(\tau)/\hbar]A_j$. With these replacements we get for the generating functional

$$Z_{\text{SET}}[\xi_1, \xi_2] = \int \mathcal{D}[\varphi] \exp \left\{ -\frac{1}{\hbar} S_{\text{SET}}[\varphi, \xi_1, \xi_2] \right\}, \quad (98)$$

where the effective action reads

$$S_{\text{SET}}[\varphi, \xi] = S_{\text{SET}}^C[\varphi] + S_1^T[\varphi, \xi_1] + S_2^T[\varphi, \xi_2]. \quad (99)$$

The first term on the rhs

$$S_{\text{SET}}^C[\varphi] = \int_0^{\hbar\beta} d\tau \left[\frac{\hbar^2 \dot{\varphi}^2(\tau)}{4E_C} + i\hbar n_g \dot{\varphi}(\tau) \right] \quad (100)$$

describes Coulomb charging of the island in presence of an applied gate voltage. The effective tunneling action

$$S_j^T[\varphi, \xi_j] = -\frac{G_j}{G_K} \int_0^{\hbar\beta} d\tau \int_0^{\hbar\beta} d\tau' \alpha(\tau - \tau') \left[1 - i\frac{e}{\hbar} \xi_j(\tau) \right] \times \left[1 + i\frac{e}{\hbar} \xi_j(\tau') \right] e^{i[\varphi(\tau) - \varphi(\tau')]} \quad (101)$$

describes quasi-particle tunneling through junction j with the kernel $\alpha(\tau)$ given by equation (25). For vanishing auxiliary field $\xi_j = 0$, the action reduces to the single electron box action [36] $S_{\text{SET}}[\varphi] = S_{\text{SET}}[\varphi, 0, 0] = S_{\text{box}}$ depending only on the parallel conductance $G_{\parallel} = G_1 + G_2$. One has

$$S_1^T[\varphi, 0] + S_2^T[\varphi, 0] = 2\frac{G_{\parallel}}{G_K} \int_0^{\hbar\beta} d\tau \int_0^{\hbar\beta} d\tau' \alpha(\tau - \tau') \sin^2 \left[\frac{\varphi(\tau) - \varphi(\tau')}{2} \right]. \quad (102)$$

Thus, the generating functional at vanishing auxiliary fields gives the box partition function $Z_{\text{SET}} = Z_{\text{SET}}[0, 0] = Z_{\text{box}}$. Performing the variational derivatives explicitly, we get for the correlator

$$\langle I_j(\tau) I_{j'}(\tau') \rangle = \langle I_j(\tau) I_j(\tau') \rangle^E \delta_{j,j'} + \langle I_j(\tau) I_{j'}(\tau') \rangle^F. \quad (103)$$

Since the auxiliary fields are in the argument of an exponential, there are two contributions. The first term comes from the second order variational derivative of the action and reads

$$\langle I_j(\tau)I_j(\tau') \rangle^E = 4\pi G_j \alpha(\tau - \tau') \frac{1}{Z_{\text{SET}}} \times \int \mathcal{D}[\varphi] \exp \left\{ -\frac{1}{\hbar} S_{\text{SET}}[\varphi] \right\} \cos[\varphi(\tau) - \varphi(\tau')]. \quad (104)$$

The second term in equation (103) involves a multiplication of two current functionals arising as first order variational derivatives of the action

$$\langle I_j(\tau)I_{j'}(\tau') \rangle^F = \frac{G_j G_{j'}}{G_K^2} \frac{1}{Z_{\text{SET}}} \times \int \mathcal{D}[\varphi] \exp \left\{ -\frac{1}{\hbar} S_{\text{SET}}[\varphi] \right\} I[\varphi, \tau] I[\varphi, \tau'], \quad (105)$$

with the current functional

$$I[\varphi, \tau] = \frac{2e}{\hbar} \int_0^{\hbar\beta} d\tau' \alpha(\tau - \tau') \sin[\varphi(\tau) - \varphi(\tau')]. \quad (106)$$

Taking into account that $\langle I_j(\tau)I_j(\tau') \rangle^E/G_j$ and $\langle I_j(\tau)I_{j'}(\tau') \rangle^F/G_j G_{j'}$ depend only on the parallel conductance $G_{\parallel} = G_1 + G_2$ and thus are independent of the indices j and j' , the conductance may be written as

$$G = \epsilon_1 \kappa_1 (G_1 E + G_1^2 F) - (\epsilon_1 \kappa_2 + \epsilon_2 \kappa_1) G_1 G_2 F + \epsilon_2 \kappa_2 (G_2 E + G_2^2 F), \quad (107)$$

where

$$E = \lim_{\omega \rightarrow 0} \frac{1}{\hbar\omega} \text{Im} \lim_{i\nu_l \rightarrow \omega + i\delta} \int_0^{\hbar\beta} d\tau e^{i\nu_l \tau} \frac{\langle I_1(\tau)I_1(0) \rangle^E}{G_1} \quad (108)$$

and

$$F = \lim_{\omega \rightarrow 0} \frac{1}{\hbar\omega} \text{Im} \lim_{i\nu_l \rightarrow \omega + i\delta} \int_0^{\hbar\beta} d\tau e^{i\nu_l \tau} \frac{\langle I_1(\tau)I_1(0) \rangle^F}{G_1^2}. \quad (109)$$

Since the conductance does not depend on the specific choice of the parameters ϵ_j and κ_j , we then find that

$$G_{\text{SET}} = G_{\text{cl}} E, \quad (110)$$

with the classical series conductance

$$G_{\text{cl}} = \frac{G_1 G_2}{G_1 + G_2}. \quad (111)$$

Within the applicability of the model this is a formally exact expression for the linear dc conductance. To proceed, we make explicit the sum over winding numbers k of the phase and write the correlator (104) in the form

$$\langle I_1(\tau)I_1(0) \rangle^E/G_1 = 4\pi\alpha(\tau) \frac{1}{Z_{\text{SET}}} \sum_{k=-\infty}^{\infty} \int_{\varphi(0)=0}^{\varphi(\hbar\beta)=2\pi k} \mathcal{D}[\varphi] \times \exp \left\{ -\frac{1}{\hbar} S_{\text{SET}}[\varphi] \right\} \cos[\varphi(\tau) - \varphi(0)]. \quad (112)$$

This result may be used as a starting point for analytical work and/or numerical calculations [51]. For further analysis, here we consider the semiclassical approximation.

5.2 Semiclassical limit

For given winding number k , the path integral may be evaluated approximately by expanding around the classical paths $\check{\varphi}^{(k)}(\tau) = \varphi^0 + \nu_k \tau$. An arbitrary path of winding number k may be written $\varphi(\tau) = \check{\varphi}^{(k)}(\tau) + \zeta(\tau)$ with $\zeta(0) = \zeta(\hbar\beta) = 0$. In terms of the Fourier coefficients $\zeta(\nu_n)$ the action reads

$$S_{\text{SET}}[\check{\varphi}^{(k)} + \zeta] = 2\pi i k \hbar m_g + S_{\text{SET}}^{(k)}[\zeta] + \sum_{m=2}^{\infty} \delta^m S_{\text{SET}}^{(k)}[\zeta], \quad (113)$$

where the first term on the rhs is the topological contribution and

$$S_{\text{SET}}^{(k)} = \hbar \left(\frac{\pi^2 k^2}{\beta E_C} + |k| \frac{g}{2} \right) \quad (114)$$

the classical action of winding number k , with the dimensionless parallel conductance $g = G_{\parallel}/G_K$. The second order variational action

$$\delta^2 S_{\text{SET}}^{(k)} = \hbar \sum_{n=1}^{\infty} \lambda_{\text{SET}}^{(k)}(\nu_n) \left| \tilde{\zeta}(\nu_n) \right|^2 \quad (115)$$

is diagonal with the eigenvalues

$$\lambda_{\text{SET}}^{(k)}(\nu_n) = \frac{2\pi^2 n^2}{\beta E_C} + g\Theta(\bar{n} - |k|)(\bar{n} - |k|). \quad (116)$$

The higher order terms in (113) read

$$\delta^{(2m+1)} S_{\text{SET}}^{(k)} = g \frac{(-1)^m}{(2m+1)!} \times \int_0^{\hbar\beta} d\sigma d\sigma' \alpha(\sigma - \sigma') \sin[\nu_k(\sigma - \sigma')] [\zeta(\sigma) - \zeta(\sigma')]^{2m+1} \quad (117)$$

for odd orders and

$$\delta^{(2m)} S_{\text{SET}}^{(k)} = g \frac{(-1)^{m+1}}{(2m)!} \times \int_0^{\hbar\beta} d\sigma d\sigma' \alpha(\sigma - \sigma') \cos[\nu_k(\sigma - \sigma')] [\zeta(\sigma) - \zeta(\sigma')]^{2m} \quad (118)$$

for even orders, with $m = 1, 2, \dots$. Since $\lambda_{\text{SET}}^{(k)}(\nu_n)$ is large for small βE_C , the expansion (113) about the classical path converges rapidly for high temperatures. At low temperatures $\lambda_{\text{SET}}^{(k)}(\nu_n)$ vanishes for $|n| < |k|$ and the simple semiclassical approximation breaks down. The zero modes

can be treated systematically for large g by considering quasi-classical trajectories with collective coordinates (sluggons) and fluctuations around them. This treatment, presented elsewhere [36] for the partition function of the single electron box, lies outside the scope of the present work and we proceed with the high temperature expansion. Rewriting the cosine function in equation (112) as a sum of exponentials, we may perform the path integral and get for the correlator

$$\langle I_1(\tau)I_1(0) \rangle^E/G_1 = 4\pi\alpha(\tau) \frac{1}{Z_{\text{SET}}} \sum_{k=-\infty}^{\infty} C_{|k|} e^{-2\pi i k n_g} \times \exp \left[-2 \sum_{m=1}^{\infty} \frac{1 - \cos(\nu_m \tau)}{\lambda_{\text{SET}}^{(k)}(\nu_m)} \right] \left[1 - \frac{1}{\hbar} S_4^{(k)} + \dots \right], \quad (119)$$

where the coefficients C_k read [36]

$$C_k = \frac{\Gamma(1+k_+) \Gamma(1+k_-)}{\Gamma^2(1+k) \Gamma(1+u)} e^{-S_{\text{SET}}^{(k)}}, \quad (120)$$

with $k_{\pm} = k + \frac{u}{2} \pm \frac{1}{2} \sqrt{4uk + u^2}$ and $u = g\beta E_C/2\pi^2$. The contribution of the third order variational action cancels, thus the dominant correction to the semiclassical approximation stems from the fourth order term

$$S_4^{(k)} = \frac{1}{2} g \hbar \beta \sum_{\substack{m,l=-\infty \\ m,l \neq 0}}^{\infty} \frac{\tilde{\alpha}(\nu_k) - 2\tilde{\alpha}(\nu_{l+k}) - 2\tilde{\alpha}(\nu_{l-k}) + \tilde{\alpha}(\nu_{m+l+k}) + \tilde{\alpha}(\nu_{m+l-k})}{\lambda_{\text{SET}}^{(k)}(\nu_m) \lambda_{\text{SET}}^{(k)}(\nu_l)}. \quad (121)$$

The corresponding expansion of the partition function Z_{SET} reads

$$Z_{\text{SET}} = \sum_{k=-\infty}^{\infty} C_{|k|} e^{2\pi i k n_g} \left[1 - \frac{1}{\hbar} S_4^{(k)} + \dots \right], \quad (122)$$

with the same correction (121). The expansions (119) and (122) proceed in powers of βE_C , however, terms involving $u = g\beta E_C/2\pi^2$ are kept to *all* orders. This ensures a meaningful result in the limit of moderately high temperatures also for large parallel conductance g .

From equation (119) one obtains for the Fourier coefficients

$$E(\nu_n) = \int_0^{\hbar\beta} d\tau e^{i\nu_n \tau} \langle I_1(\tau)I_1(0) \rangle^E/G_1 \quad (123)$$

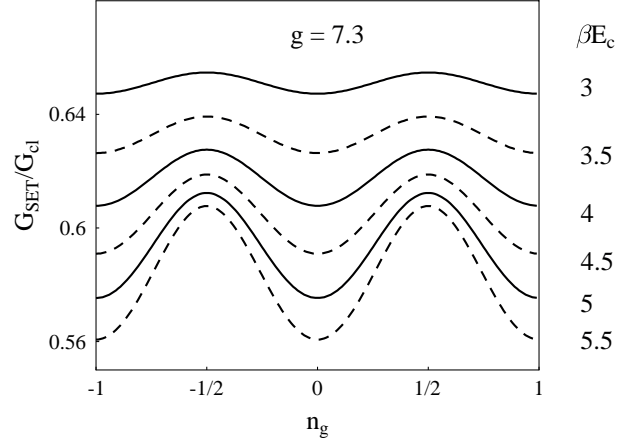


Fig. 13. Conductance of a symmetrical SET in dependence on the dimensionless gate voltage n_g for various temperatures and parallel conductance $g = 7.3$.

the high temperature expansion

$$E(\nu_n) = \frac{4\pi}{Z_{\text{SET}}} \sum_{k=-\infty}^{\infty} C_{|k|} e^{-2\pi i k n_g} \times \exp \left[-2 \sum_{l=1}^{\infty} \frac{1}{\lambda_{\text{SET}}^{(k)}(\nu_l)} \right] \left\{ \tilde{\alpha}(\nu_{n+k}) + \sum_{m \neq 0} \frac{\tilde{\alpha}(\nu_{n+k+m})}{\lambda_{\text{SET}}^{(k)}(\nu_m)} + \frac{1}{2} \sum_{m,l \neq 0} \frac{\tilde{\alpha}(\nu_{n+k+m+l})}{\lambda_{\text{SET}}^{(k)}(\nu_m) \lambda_{\text{SET}}^{(k)}(\nu_l)} - \frac{1}{\hbar} \tilde{\alpha}(\nu_{n+k}) S_4^{(k)} + \mathcal{O}(\beta E_C)^3 \right\}. \quad (124)$$

Since no $1/\lambda_{\text{SET}}^{(k)}(\nu_n)$ term appears, the order of the expression remains the same after analytical continuation. When $E(\nu_n)$ is analytically continued in the complex ν plane, $E(\nu)$ is analytic on each half plane $\text{Re } \nu \leq 0$ with a cut along the imaginary axis [46]. The representation of $E(\nu)$ as a sum over winding numbers k shifts this cut to $\text{Re } \nu = k$ for the k th term of the sum. Thus, in the phase representation, only the full sum shows the analytic properties underlying the conductance formula (8). The summands of the high temperature expansion (124) for winding number k are of the form $g(k)f(|n+k|) = \Theta(-n-k)g(k)f(-n-k) + \Theta(n+k)g(k)f(n+k)$, where $f(n), g(n)$ are analytic functions and $g(k) = g(-k)$.

Now, the sum over winding numbers may be expressed as a sum over charges m

$$\sum_{k=-\infty}^{\infty} g(k)f(|n+k|) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{-n} d\kappa e^{2\pi i m \kappa} g(\kappa)f(-n-\kappa) + \int_{-n}^{\infty} d\kappa e^{2\pi i m \kappa} g(\kappa)f(n+\kappa). \quad (125)$$

Performing the limits $i\nu_n \rightarrow \omega + i\delta$ and $\omega \rightarrow 0$ for the rhs of this equation and rewriting the result again as a sum over winding numbers, we obtain

$$\lim_{\omega \rightarrow 0} \frac{1}{\hbar\omega} \text{Im} \lim_{i\nu_n \rightarrow \omega + i\delta} \sum_{k=-\infty}^{\infty} g(k) f(|n+k|) = -\frac{\hbar\beta}{2\pi} \sum_{k=-\infty}^{\infty} g(k) \frac{\partial}{\partial k} f(|k+\delta|) = -\frac{\hbar\beta}{2\pi} g(0) f'(0). \quad (126)$$

The sums in equation (124) may be performed exactly with the help of integral representations of the psi function and its derivatives [52]

$$\psi(z) = \int_0^{\infty} dt \left(\frac{e^{-t}}{t} - \frac{e^{-zt}}{1-e^{-t}} \right) \quad (127)$$

and

$$\psi^{(n)}(z) = (-1)^{n+1} \int_0^{\infty} dt \frac{t^n e^{-zt}}{1-e^{-t}}. \quad (128)$$

This way the high temperature expansion of the conductance may be evaluated to read

$$G_{\text{SET}} = G_{\text{cl}} Z_{\text{SET}}^{-1} \exp \{ -2[\gamma + \psi(1+u)]/g \} \times \{ 1 - \psi'(1+u)(\beta E_C/\pi^2) + [g\sigma(u) + \tau(u)] (\beta E_C/2\pi^2)^2 + \mathcal{O}(\beta E_C)^3 \}. \quad (129)$$

The dependence on $u = g\beta E_C/2\pi^2$ is given in terms of two auxiliary functions

$$\sigma(u) = \frac{\gamma + \psi(1+u) - u\psi'(1+u)}{u^2} + \int_0^1 dv \frac{2v(1-v^u)\phi(v, 1, 1+u)}{(1-v)u} \quad (130)$$

and

$$\tau(u) = -\frac{3\gamma\psi'(1+u) + \psi(1+u)[\frac{\pi^2}{6} + 2\psi'(1+u)]}{u} - \frac{[\psi(1+u) + \gamma]^2}{u^2} + \frac{\pi^2}{6u}\psi(1+u) + \int_0^1 dv \Xi(u, v), \quad (131)$$

with Lerch's transcendent $\phi(z, u, v)$ [53] and

$$\Xi(u, v) = \frac{\psi'(1+u)}{u \ln(v)} + \frac{1}{(1-v)u} \left\{ 2v\phi(v, 2, 1+u) + \frac{1-2v^u}{v} \left[\ln(v)\phi\left(\frac{1}{v}, 1, 1+u\right) + \phi\left(\frac{1}{v}, 2, 1+u\right) \right] + v^u \left[2\ln(v)\ln(1-v) + \frac{1}{2}\ln^2(v) + 3\text{Li}_2(1-v) \right] - \frac{2(1-v^u)}{u} [\ln(1-v) + v\phi(v, 1, 1+u)] \right\}. \quad (132)$$

The high temperature expansion of Z is straightforward and reads

$$Z_{\text{SET}} = 1 + g\sigma(u)(\beta E_C/2\pi^2)^2 + \mathcal{O}(\beta E_C)^3 + 2C_1 \cos(2\pi n_g) [1 + \mathcal{O}(\beta E_C)^2], \quad (133)$$

which combines with equation (129) to yield an analytical expression for the high temperature conduction of a SET valid for arbitrary tunneling conductance.

5.3 Discussion of results and comparison with experimental data

In Figure 13 the normalized conductance $G_{\text{SET}}/G_{\text{cl}}$ is depicted in dependence on the dimensionless gate voltage n_g for various temperatures βE_C . The quantum corrections are more pronounced for lower temperatures where the gate voltage dependence becomes more significant. The oscillatory behavior of the conductance may be characterized in terms of a maximum $G_{\text{max}} = G_{\text{SET}}|_{n_g=1/2}$ and minimum $G_{\text{min}} = G_{\text{SET}}|_{n_g=0}$ linear conductance. We have compared our findings for the maximum and minimum as a function of temperature with recent experimental data by Joyez *et al.* [16] for transistors with $g = 0.6, 2.5$ and 7.3 . As seen from Figure 14 the theory describes the high temperature behavior of all junctions (results for $g = 0.6$ are not shown) down to temperatures where the current starts to modulate with the gate voltage. The parameters have *not* been adjusted to improve the fit but coincide with the values given in [16]. A recent Monte-Carlo study [51] covering the entire range of temperatures has shown that the small deviations between theory and experiment for $g = 7.3$ near $\beta E_C = 1$ arise from uncertainties in the parameter g . The estimate $g = 7.3$ based on the assumption of a symmetrical transistor should be replaced by a value near $g = 10$. We mention that the temperature dependence of the conductance of the highly conducting SET is not within reach of previous theoretical work. The results obtained should be useful for experimental studies of even larger tunneling conductances since the predictions remain valid for arbitrary values of g .

In the region of weak tunneling, $g < 1$, the quantity u becomes small at high temperatures and we may replace $\sigma(u)$ and $\tau(u)$ by

$$\sigma(0) = 6\zeta(3), \quad \tau(0) = \pi^4/10. \quad (134)$$

This gives for the conductance of a weakly conducting SET

$$\frac{G_{\text{SET}}}{G_{\text{cl}}} = \left[1 - \frac{\beta E_C}{3} + \left(\frac{1}{15} + g \frac{3\zeta(3)}{2\pi^4} \right) \times (\beta E_C)^2 + \mathcal{O}(\beta E_C)^3 \right], \quad (135)$$

in accordance with earlier work [54–56]. In the region of strong tunneling, the quantity u is typically large even for the highest temperatures explored experimentally and the full expression (129, 133) must be used.

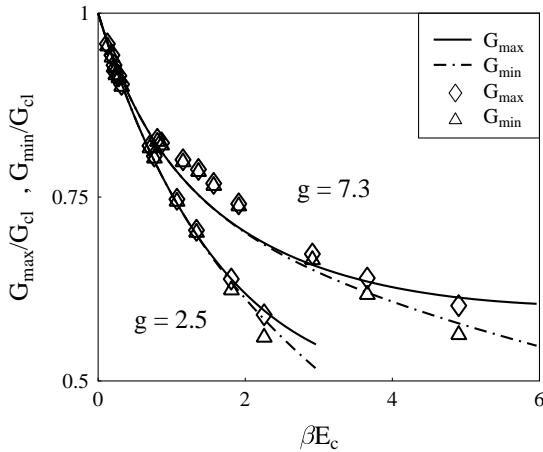


Fig. 14. Maximum and minimum linear conductance in dependence on dimensionless temperature for two dimensionless parallel conductances $g = 2.5$ and 7.3 compared with experimental data (symbols) by Joyez *et al.* [16].

6 Conclusions

In this article we have studied the conductance of nanofabricated metallic circuits showing Coulomb blockade phenomena. We have treated electron tunneling nonperturbatively based on a path integral expression derived in Section 2. Then, the frequency dependent linear conductance of a single tunnel junction embedded in an electromagnetic environment was calculated in the semiclassical approximation. We have shown that this approximation is not only adequate for high temperatures but also in the limit of large conductance. As far as the leading quantum corrections are concerned, the tunnel junction was shown to be described as an effective linear element with an admittance that depends on the whole circuit. The predictions for the dc conductance were compared with recent experimental findings by two groups [17,18] and we found good agreement in the semiclassical regime of large conductance and/or high temperatures. Further, we have shown that the low frequency behavior of the ac conductance can be calculated in terms of a renormalized capacitance which shows a linear dependence on the inverse temperature.

In Section 4 we applied the method to a linear array of N tunnel junctions and determined the effect of the environmental impedance on the conductance as well as the influence of the array length N . For large N the conductance dip becomes independent of the electromagnetic environment in accordance with previous calculations [49]. For multi-junction circuits the configuration space of the phase variables was shown to be a torus, and contributions of nonvanishing winding numbers become relevant if one goes beyond the leading order quantum corrections.

The conductance of the single electron transistor was determined in Section 5 by including nontrivial winding numbers leading to the gate voltage dependence of the conductance. The results were found to explain recent experimental data [16] for moderately low temperatures. For

lower temperatures the contribution of sluggish trajectories [36] has to be taken into account which was not elaborated here.

The semiclassical theory presented has features in common with the quasiclassical Langevin equation put forward in reference [35]. The Gaussian approximation underlying this approach is consistent with the semiclassical theory up to first order in βE_C . Our results consistently include higher order terms in βE_C for arbitrary tunneling conductances. Non-Gaussian fluctuations are particularly relevant in the moderately large tunneling regime. The analytical theory presented covers one edge in the temperature/conductance plane, arbitrary conductance and sufficiently high temperatures. Another edge is described by the perturbative approach, arbitrary temperature and sufficiently small tunneling conductance. Both theories arise naturally from the formally exact representation of the current-current correlator which may also serve as a basis for Monte-Carlo simulations [51] that bridge between the semiclassical and perturbative results.

The authors would like to thank Michel Devoret, Daniel Esteve, Shadyar Farhangfar, Philippe Joyez, Jukka Pekola, and Jussi Toppari for valuable discussions. One of us (GG) acknowledges the hospitality of the CEA-Saclay during an extended stay. Financial support was provided by the Deutsche Forschungsgemeinschaft (DFG) and the Deutscher Akademischer Austauschdienst (DAAD).

References

1. *Single Charge Tunneling*, edited by H. Grabert, M.H. Devoret, NATO ASI Series B (Plenum, New York, 1992), Vol. 294.
2. T.A. Fulton, G.J. Dolan, Phys. Rev. Lett. **59**, 109 (1987).
3. P. Delsing, K.K. Likharev, L.S. Kuzmin, T. Claeson, Phys. Rev. Lett. **63**, 1180 (1989).
4. M.A. Kastner, Rev. Mod. Phys. **64**, 849 (1992).
5. J.P. Kauppinen, J.P. Pekola, Phys. Rev. Lett. **77**, 3889 (1996).
6. M.H. Devoret, D. Esteve, H. Grabert, G.-L. Ingold, H. Pothier, C. Urbina, Phys. Rev. Lett. **64**, 1824 (1990).
7. D.V. Averin, K.K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B.L. Altshuler, P.A. Lee, R.A. Webb (North-Holland, Amsterdam, 1991), p. 173.
8. A.A. Odintsov, G. Falci, G. Schön, Phys. Rev. B **44**, 13089 (1991).
9. G.-L. Ingold, Yu.V. Nazarov, in reference [1].
10. Yu.V. Nazarov, Phys. Rev. B **43**, R6220 (1991).
11. P. Lafarge, H. Pothier, E.R. Williams, D. Esteve, C. Urbina, M.H. Devoret, Z. Phys. B **85**, 327 (1991).
12. A.N. Cleland, D. Esteve, C. Urbina, M.H. Devoret, Appl. Phys. Lett. **61**, 2820 (1992).
13. J.P. Pekola, K.P. Hirvi, J.P. Kauppinen, M.A. Paalanen, Phys. Rev. Lett. **73**, 2903 (1994).
14. D.V. Averin, Yu.V. Nazarov, in reference [1].
15. J. König, H. Schoeller, G. Schön, Phys. Rev. Lett. **78**, 4482 (1997).
16. P. Joyez, V. Bouchiat, D. Esteve, C. Urbina, M.H. Devoret, Phys. Rev. Lett. **79**, 1349 (1997).

17. P. Joyez, D. Esteve, M.H. Devoret, Phys. Rev. Lett. **80**, 1956 (1998).
18. Sh. Farhangfar, J.J. Toppari, Yu.A. Pashkin, A.J. Manninen, J.P. Pekola, Europhys. Lett. **43**, 59 (1998).
19. D. Chouvaev, L.S. Kuzmin, D.S. Golubev, A.D. Zaikin, Phys. Rev. B **59**, 10599 (1999).
20. H. Grabert, Phys. Rev. B **50**, 17364 (1994).
21. H. Grabert, Physica B **194-196**, 1011 (1994).
22. G. Göppert, H. Grabert, N.V. Prokof'ev, B.V. Svistunov, Phys. Rev. Lett. **81**, 2324 (1998).
23. J. König, H. Schoeller, G. Schön, Phys. Rev. B **58**, 7882 (1998).
24. G. Göppert, H. Grabert, C. Beck, Europhys. Lett. **45**, 249 (1999).
25. K.A. Matveev, Sov. Phys. JETP **72**, 892 (1991).
26. H. Schoeller, G. Schön, Phys. Rev. B **50**, 18436 (1994).
27. G. Falci, G. Schön, G.T. Zimanyi, Phys. Rev. Lett. **74**, 3257 (1995).
28. D.S. Golubev, A.D. Zaikin, Phys. Rev. B **50**, 8736 (1994).
29. J. König, H. Schoeller, Phys. Rev. Lett. **81**, 3511 (1998).
30. T. Pohjola, J. König, H. Schoeller, G. Schön, Phys. Rev. B **59**, 7579 (1999).
31. G. Schön, A.D. Zaikin, Phys. Rep. **198**, 237 (1990).
32. E. Ben-Jacob, E. Mottola, G. Schön, Phys. Rev. Lett. **51**, 2064 (1983).
33. S.V. Panyukov, A.D. Zaikin, Phys. Rev. Lett. **67**, 3168 (1991).
34. D.S. Golubev, A.D. Zaikin, Phys. Rev. B **46**, 10903 (1992).
35. D.S. Golubev, A.D. Zaikin, JETP Lett. **63**, 1007 (1996).
36. X. Wang, H. Grabert, Phys. Rev. B **53**, 12621 (1996).
37. X. Wang, R. Egger, H. Grabert, Europhys. Lett. **38**, 545 (1997).
38. W. Hofstetter, W. Zwerger, Phys. Rev. Lett. **78**, 3737 (1997).
39. C.P. Herrero, G. Schön, A.D. Zaikin, Phys. Rev. B **59**, 5728 (1999).
40. G. Göppert, X. Wang, H. Grabert, Phys. Rev. B **55**, R10213 (1997).
41. G. Göppert, H. Grabert, Phys. Rev. B **58**, R10155 (1998).
42. G. Göppert, H. Grabert, C. R. Acad. Sci. **327**, 885 (1999).
43. G.J. Dolan, J.H. Dunsmuir, Physica B **152**, 7 (1988).
44. J. Bardeen, Phys. Rev. Lett. **6**, 57 (1961).
45. A.O. Caldeira, A.J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).
46. G. Baym, N.D. Mermin, J. Math. Phys. **2**, 232 (1961).
47. H. Grabert, P. Schramm, G.-L. Ingold, Phys. Rep. **168**, 115 (1988).
48. G. Göppert, H. Grabert (to be published).
49. Sh. Farhangfar, K.P. Hirvi, J.P. Kauppinen, J.P. Pekola, J.J. Toppari, D.V. Averin, A.N. Korotkov, J. Low Temp. Phys. **108**, 191 (1997).
50. Sh. Farhangfar, A.J. Manninen, J.P. Pekola, Europhys. Lett. **49**, 237 (2000).
51. G. Göppert, B. Hüpper, H. Grabert (to be published).
52. M. Abramowitz, I.A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1970).
53. W. Magnus, F. Oberhettinger, R.P. Soni, *Formulas and Theorems for the Special Functions of Mathematical Physics* (Springer, Berlin, 1966).
54. G. Göppert, Diploma thesis, Freiburg, 1996.
55. P. Joyez, D. Esteve, Phys. Rev. B **56**, 1848 (1997).
56. D.S. Golubev, J. König, H. Schoeller, G. Schön, A.D. Zaikin, Phys. Rev. B **56**, 15782 (1997).